A SWITCHING MODEL APPROACH TO STOCK PRICE MODELING

HANDE ORUÇ

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İSTANBUL BİLGİ ÜNİVERSİTESİ

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Toplam Sayfa Sayısı:

Anahtar Kelimeler (İngilizce)	Anahtar Kelimeler (Türkçe)
1.Regime Switching Model	1.Rejim Değiştirme Modelleri
2.Geometric Brownian Motion	2.Geometrik Brown Devinimi
3.Stock Prices	3.Hisse Senedi Fiyatları

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Abstract

In this study, we consider a nonlinear probabilistic discretized version of Geometric Brownian Motion (GBM) to model the stock prices traded in Istanbul Stock Exchange. By nonlinearity we mean the existence of different states in the model, namely positive return process, negative return process. As the names imply, each process is formed using positive and negative returns respectively. The model decides which process to use according to a probabilistic framework endogenously determined in the model. By means of these probabilities, this model is designed to give better fit than GBM, where the better fit is acquired by Mean Squared Errors (MSE). We obtain the results via the Monte Carlo technique using Matlab and hundred stock prices. As a result, the obtained probabilities after simulation demonstrate that positive returns tend to be followed by negative return process and vice versa.

Özet

Bu çalışmada, Istanbul Menkul Kıymetler Borsası'nda işlem gören hisse senetlerini doğrusal olmayan olasılıklı ayrıklaştırılmış Geometrik Brownian Devinimini kullanarak modellendi. Hisse senetlerini pozitif getiri ve negatif getiri olarak ikiye ayırarak doğrusal olmayan model oluşturuldu. Model içerisinde belirlenen olasılıklarla positif veya negatif getiri süreçleri kullanılarak model oluşturuldu. Olasılıklar hata karelerinin ortalamları minimum olacak şekilde belirlendi. Sonuçlar, MATLAB yardımıyla IMKB'de işlem gören yüz hisse senedi için Monte Carlo simulasyonu yapılarak bulundu. Yapılan çalışmanın sonucunda; pozitif getirilerin negatif getirileri, negatif getirilerin ise pozitif getirileri takip ettiği ortaya çıktı.

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Section 1

Introduction

With the progress of financial markets, modeling stock prices became very popular. Stock price data are modeled in two different types: discrete and continuous time processes. The analysis related to the processes which are examined as discrete time are called time series analysis and in the time domain the simplest time series models such as random walk, autoregressive, moving average models etc. are based on regression analysis. Since many relationships in finance are nonlinear, these linear models are unable to capture the features of the data in finance.

Therefore, two different types of nonlinear models are formed. On the one hand, stochastic variable models such as ARCH, GARCH models are used for modeling and forecasting volatility; on the other hand, deterministic variables models such as regime switching models allow the behaviour of a series to follow different processes at different points in time.

In recent years, regime switching models which use thresholds have become popular in finance. As an example, we can separate the state of market into two different regimes such as "bullish" and "bearish". The special feature of these models is that the key parameters such as mean, volatility etc. tend to change according to the state of the market. Furthermore, in a regime switching model, the state of the related market is divided into a finite number of regimes. Hence, the assumption of linearity in time series analysis has been abandoned and the study of nonlinear model becomes increasingly popular. Tong (1978) has developed threshold autoregressive model (TAR) which is one class of nonlinear model. The property of this model is that the process is divided into regimes and for each regime there are different AR models. In addition to

TAR model, Markov switching model which is developed by Hamilton (1989) posits that regime switches are exogenous according to probabilities. The regime switching models are not designed to explain the reason of the regime changes occurance and the timing of such changes. In addition to ARCH model and Markov switching model, Hamilton and Susmel (1994) proposed the SWARCH model, a mixture of Markov-switching model and ARCH model. SWARCH model assumes different parameter values in the conditional variance equation.

All models that we mentioned above are used to estimate discrete-time stochastic process, in which the price changes at discrete time points. In financial markets, however, assets are traded at very frequent intervals of time. A reasonable approximation is to let the interval of time go arbitrarily close to zero, which leads to a world of continuous-time models. In addition to that, the efficient market hypothesis suggests that share price should follow random walks. The continuous-time analogues of a random walk and a random walk with drift are Brownian Motion(BM) and Geometric Brownian Motion (GBM), respectively.

After BM having been invented, it has been used by the vast branch of science such as finance, engineering etc. BM which is common to have a continuous time price process modeled in finance was discovered by the English botanist Robert Brown in 1827. In the process of time, different types of BM are composed. For example, Phadke and Wu (1974) developed continuous time autoregressive models (CAR).

The models which are used to model discrete time series can be adapted to continuous time finance. As ARCH and GARCH are used to model volatility in discrete time series, a family of continuous-time generalized autoregressive conditionally heteroscedastic process, generalizing the COGARCH process of Klüppelberg et *al.* (2004), is introduced and studied. Additionally, Brockwell et *al.* (2006) developed this continuous time GARCH process and the volatility process is found to have autocorrelation function of a continuous-time autoregressive moving average.

As TAR model was invented in discrete time by using deterministic variable autoregressive model, it has been adapted to nonlinear GBM, as well. For this purpose, Tong and Yeung (1991), Brockwell et al. (1991) developed continuous time threshold autoregressive model (CTAR). On the other part, Gerber and Shiu (2003) and Jose(2008) tried to constitute GBM which is developed by a barrier strategy. According to this strategy assets are modeled to assume that there exists upper and lower barriers and the regime tends to change thanks to these barriers. There are two different μ and σ in this model and the first regime continues until it hits the upper threshold. Then the regime changes and the process follows second regime and it maintains the second regime until it hits negative threshold.

In addition to these models, Markov switching model(MSM) has been evolved. Guo (2001) considered the problem of pricing Russian options which are look-back option. Based on the structure of Russian options, he model stock prices by using the pair (μ, σ) takes different values for different states. Buffington and Elliott (2002) generalize American option with regime switching. The contribution of them is continuous time Markov switching model which is widely used in finance. Additionally, Yin et al. (2006) assumed that stock prices follow continuous time Markov switching model and tried to find optimum stopping time for American put option.

In this study, we form a mixed model which is a probabilistic threshold discretized version of GBM for stock prices which are traded in Istanbul Stock Exchange. Our discretized version of modified GBM which is designed to give better fit than discretized GBM according to Mean Squared Error (MSE) is formed of two different process as positive return path and negative return path. If logreturn of stock prices is positive at time t, then the process follows positive return path with probability p or negative return path with probability $1 - p$ at time $t+1$. On the other hand, if logreturn of stock prices is negative at time t, then the process follows negative return path with probability q or positive return path with probability $1-q$ at time $t+1$. The results which we obtain via Monte Carlo technique using Matlab demonstrate that the stock prices tend to follow negative return path at time $t+1$ if the logreturn of stock prices is positive at time t and vice versa. The thesis is organized as follows: Section 2 mentions data sets that we used during this study. Section 3, gives basic information related to TAR & CTAR, Markov switching model & continuous-time Markov switching model and GBM, respectively. Section 4 explains the discretized verion of modified GBM and methodology, Section 5 gives results. Section 6 includes conclusion and future work.

Section 2

The Data

During this study we use hundred stock prices from Istanbul Stock Exchange Market. The date interval of these price data are between 2003 and 2009; however, some of them have shorter interval than others. The source of these data is FOREKS Bilgi İletişim Hizmetleri A.Ş. (The list of these stock prices is attached to the Appendix.)

The histogram and descriptive statistics of ISE100 which covers the hundred stock prices is as the following;

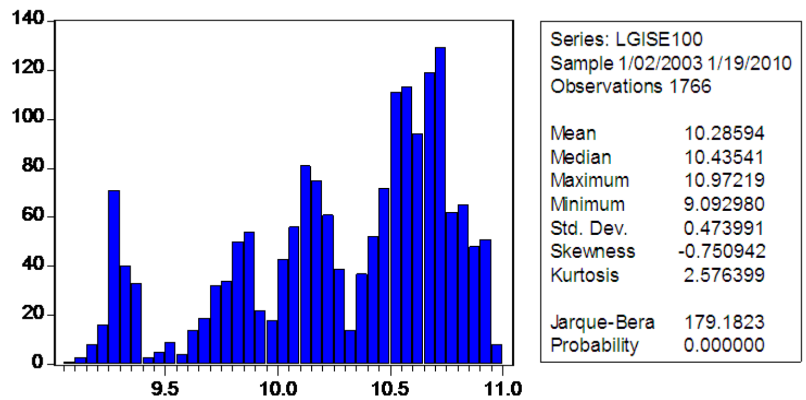

Figure 1: Histogram and Descriptive Statistics of log(ISE100)

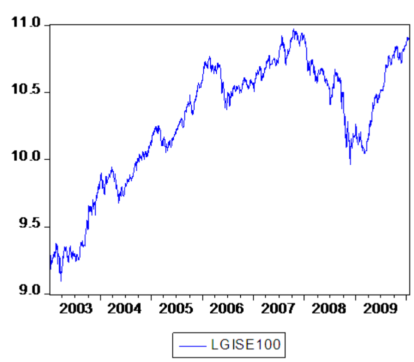

Figure 2: Graph of log(ISE100)

Section 3.1

Threshold Autoregressive Model (TAR) & Continuous Time Threshold Autoregressive Model (CTAR)

As we mention above, threshold autoregressive model (TAR) exhibits the nonlinearity of time series data. It means that if it is anticipated a change of regime's presence, regime switching models are used. TAR which is one of the most popular regime switching model seperates from autoregressive model by using two different slope coefficients such as α_1 where the sequence $y_{t-1} > 0$ and α_2 where $y_{t-1} \leq 0$. Then the model is as follows:

$$y_t = \left\{ \begin{array}{l} \alpha_1 y_{t-1} + \varepsilon_{1t} \text{ if } y_{t-1} > 0 \\ \alpha_2 y_{t-1} + \varepsilon_{2t} \text{ if } y_{t-1} \leq 0 \end{array} \right\} \tag{1}$$

Eventhough the sequence y_t seems linear when $y_{t-1} > 0$ or $y_{t-1} \leq 0$, the entire y_t is nonlinear. The error terms ε_{1t} and ε_{2t} have the occurance of two different regimes provided. We mean that if y_{t-1} is positive and close to zero,a negative value of ε_{1t} will cause to change the regime or vice versa. In addition to that if we assume that the variance of ε_{1t} is larger than the variance of ε_{2t}, the probability of switching from positive regime to negative regime is higher than from negative regime to positive regime. On the other hand, if we assume that the two error terms have equal variances then the equation can be written as follows:

$$y_t = \alpha_1 I_t y_{t-1} + \alpha_2 (1 - I) y_{t-1} + \varepsilon_t \tag{2}$$

where $I_t = 1$ if $y_{t-1} > 0$ and $I_t = 0$ if $y_{t-1} \leq 0$.

Before studying continuous time threshold autoregressive (CTAR) process, we quickly want to give basic notation of continuous time autoregressive model. We define a zero mean CAR(1) process.

$$dY_t = aY_t dt + \sigma dW_t \qquad -\infty < t < \infty \tag{3}$$

Now, we can define continuous time threshold autoregressive model in the paper of Brockwell, Hyndman and Grundwald (1991). We define the CTAR(1) process as follows;

$$dY_t = (a^{(i)} Y_t + b^{(i)})dt + \sigma^{(i)} dW_t \qquad r_{i-1} < X_t < r_i \quad i = 1, ..., d \tag{4}$$

where -$\infty = r_0 < r_1 < ... < r_l = \infty$, $a^{(i)} < 0$,each $\sigma^{(i)} > 0$ and $b^{(1)}, b^{(2)}, ..., b^{(d)}$ are constants. The threshold are the levels $r_1, r_2, r_3, ..., r_{d-1}$.If we take d=1, then $\{(Y_t)\}$ is a CAR(1) process.

Section 3.2

Markov Switching Model
&
Continuous Time Markov Switching Model

In addition to the idea of using probability switching in nonlinear time series analysis which is discussed in Tong (1983), Hamilton (1989) delevopes the Markov switching autoregressive (MSA) model. A time series y_t follows an MSA model if it satisfies

$$y_t = \left\{ \begin{array}{ll} c_1 + \sum_{i=1}^{p} \phi_{1,i} y_{t-1} + a_{1t} & \text{if s}_t\text{=1} \\ c_2 + \sum_{i=1}^{p} \phi_{2,i} y_{t-1} + a_{2t} & \text{if s}_t\text{=2} \end{array} \right\} \tag{5}$$

with transition probabilities

$$p(s_t = 2 \mid s_{t-1} = 1) = w_1, \tag{6}$$

$$p(s_t = 1 \mid s_{t-1} = 2) = w_2 \tag{7}$$

where $\{a_{1t}\}$ and $\{a_{2t}\}$ are sequence of iid random variables with mean zero and finite variance and are independent each other.Since the states are not directly observable, it is much harder to estimate an MSA model than other models.However, MSA model can easily be generalized to the case of more than two states.

The continuous time Markov switching model which is developed by Buffington and Elliott (2002) and Guo (2001) comprises of an observed process $R = (R_t)_{t\in[0,T]}$ where

$$R_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s,\ t \in [0,T] \tag{8}$$

where $(W_t)_{t\in[0,T]}$ is a standard Brownian Motion , $(\mu_t)_{t\in[o,T]}$ is the time varying drift and $(\sigma_t)_{t\in[0,T]}$ the time varying volatility. The drift process μ and the volatility process σ, taking values in R^n and R^{n*n}, respectively, are continuous time, time homogeneous, irreducible Markov process with d states, adapted to $\mathcal{F}$,independent of W,driven by the state process $Y = (Y_t)_{t\in[0,T]}$. We denote the possible values of μ and σ with $\mu^{(1)}, ..., \mu^{(d)}$ and $\sigma^{(1)}, ..., \sigma^{(d)}$, respectively. The state process Y, which is a continuous time, time homogeneous, irreducible Markov Process adapted to $\mathcal{F}$, independent of W, with state space $\{1, ...d\}$ allows for the representations

$$\mu_t = \sum_{k=1}^{d} \mu^{(k)} 1_{\{Y_t=k\}} \tag{9}$$

$$\sigma_t = \sum_{k=1}^{d} \sigma^{(k)} 1_{\{Y_t=k\}}. \tag{10}$$

Section 3.3

Geometric Brownian Motion

GBM which is a type of BM is popular in finance because of the contribution to the modelling asset price used in the Black-Scholes-Merton option pricing formula. GBM can be defined as the follows:

$$d(S_t) = \mu S_t dt + \sigma S_t dW_t \tag{11}$$

where μ is called drift, σ is called volatility, W_t is a normalized BM and S_t is a stochastic process.Moreover, both μ and σ are constants.

For an arbitrary initial value S_0 the equation has the analytic solution

$$S_t = S_0 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma W_t\right) \tag{12}$$

Using transformation and Ito's formula, we make inference the equation satisfied by the logarithm of the price. The equation will be as follows:

$$d(S_t) = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t) \tag{13}$$

The discretized version of the geometric BM is

$$\Delta \log S_t = (\mu - \frac{\sigma^2}{2})\Delta t + \sigma \Delta W_t \tag{14}$$

Let $\varepsilon_t = W_t - W_{t-1}$, and we know that $\Delta t = 1$.Then the equation can be rewritten as

$$\Delta \log S_t = \mu - \frac{\sigma^2}{2} + \sigma \varepsilon_t \tag{15}$$

μ and σ are parameters that we want to estimate.

Using maximum likelihood method, we can estimate mean and variance of $\Delta \log S_t$:

$$\widehat{\mu}_T = \widehat{m}_T + \frac{\widehat{\sigma}_T^2}{2} \tag{16}$$

$$\widehat{\sigma}_T^2 = \widehat{s}_T^2 \tag{17}$$

where $\widehat{m}_T = \frac{1}{T}\sum_{t=1}^{T}(\Delta \log S_t)$ and $\widehat{s}_T^2 = \frac{1}{T}\sum_{t=1}^{T}(\Delta \log S_t - \widehat{m}_T)^2$.[1]

[1] Gourieroux, C. and Jasiak, J. (2001). *Financial Econometrics*, Princeton University Press, Princeton.

Section 4

The Modified Geometric Brownian Motion and Methodology

As we discussed above, GBM has linear path. In real world, however, the stocks tend to fluctuate. In other words, the stocks can change their regimes. Therefore, we want to model another GBM which is nonlinear continuous time model and has been obtained a combination of threshold autoregressive model and Markov switching model. The new model that we compose is as follows:

$$dS_t = \left\{ \begin{array}{lll} \mu_1 S_t dt + \sigma_1 S_t dW_{1t} & \text{(with probability p)} & \text{if } R_t > 0 \\ \mu_2 S_t dt + \sigma_2 S_t dW_{2t} & \text{(with probability 1-p)} & \\ & & \\ \mu_2 S_t dt + \sigma_2 S_t dW_{3t} & \text{(with probability q)} & \text{if } R_t < 0 \\ \mu_1 S_t dt + \sigma_1 S_t dW_{4t} & \text{(with probability 1-q)} & \end{array} \right\} \quad (18)$$

and a discretized version, we get

$$\triangle \log S_t = \left\{ \begin{array}{lll} (\mu_1 - \frac{\sigma_1^2}{2})dt + \sigma_1 \triangle W_{1t} & \text{(with probability p)} & \text{if } R_t > 0 \\ (\mu_2 - \frac{\sigma_2^2}{2})dt + \sigma_2 \triangle W_{2t} & \text{(with probability 1-p)} & \\ & & \\ (\mu_2 - \frac{\sigma_2^2}{2})dt + \sigma_2 \triangle W_{3t} & \text{(with probability q)} & \text{if } R_t < 0 \\ (\mu_1 - \frac{\sigma_1^2}{2})dt + \sigma_1 \triangle W_{4t} & \text{(with probability 1-q)} & \end{array} \right\} \quad (19)$$

where $\mathrm{W}_t = \sqrt{t}\varepsilon$ and $\varepsilon \sim N(0,1)$, $R_t = \log S_t - \log S_{t-1}$ and $\mu_1, \mu_2, \sigma_1,\ \sigma_2, p, q$ are parameters that we want to estimate.

To estimate these parameters, we firstly seperate the dataset into two sub-datasets: one is $R_t < 0$; the other one is $R_t > 0$.Then we estimate these parameters as follows;

$$\widehat{\mu}_{iT} = \widehat{m}_{iT} + \frac{\widehat{\sigma}^2_{iT}}{2} \tag{20}$$

$$\widehat{\sigma}^2_{iT} = \widehat{s}^2_{iT} \tag{21}$$

where $\widehat{m}_{iT} = \frac{1}{T}\sum_{t=1}^{T}(R_t)$,$\widehat{s}^2_{iT} = \frac{1}{T}\sum_{t=1}^{T}(R_t - \widehat{m}_{iT})^2$, $i = 1$ for positive logreturn path and $i = 2$ for negative logreturn path.[2]

To estimate the probabilities p and q, we use Mean Squared Error (MSE) which is a statistical method. The probabilities p and q which minimize MSE of the discretized version of modified GBM are choosen as optimum p and q. MSE is one of many ways to quantify the amount by which an estimator differs from the true value of the quantity being estimated. Suppose the forecast sample is $j = T+1, T+2, ..., T+h$, and denote the actual and forecasted value in period t as y_t and $\widehat{y_t}$, respectively.

The Mean Squared Error statistic is computed as follows:

$$MSE = \frac{1}{h}\sum_{t=T+1}^{T+h}(\widehat{y_t} - y_t)^2 \tag{22}$$

Ideally, the MSE will be as small as possible. As the fit of the model improves, the MSE will approach 0. Now, we want to model the stock prices by using two different models which are discretized GBM and discretized version of modified GBM. Firstly, we assume that stock prices follow GBM defined in eq.(11) and μ and σ are calculated using eq.(16) and eq.(17). Then, using eq. (14) we estimate stock prices. However, since we have a random variable in the model, we repeat our estimation 50 times to find more realistic result . Then we take the mean of 50 MSEs as MSE of discretized GBM. Secondly, we want to reestimate stock

[2] Gourieroux, C. and Jasiak, J. (2001). *Financial Econometrics*, Princeton University Press, Princeton.

prices using our discretized version of modified GBM. That means we suppose that the stock prices follow our modified GBM defined in eq.(18). We seperate the stock price two parts as positive returns and negative returns. Using eq.(20) and eq.(21), we compute two different μ and σ. It means that there are two different process as positive and negative.

The processes which has positive path is as follows;

$$dS_t = \mu_1 S_t dt + \sigma_1 S_t dW_{1t} \tag{23}$$

and the process which has negative path is as follows;

$$dS_t = \mu_2 S_t dt + \sigma_2 S_t dW_{2t} \tag{24}$$

Now, using eq.(19), we predict stock prices, again. During the prediction, to obtain the stock prices by using the discretized version of modified GBM we should find the probabilities which minimize MSE. For this purpose, we fix $p = 0$. Then q is choosed 0 for the first step and will be increased 0.1 for the next step until it reaches 1. For each fixed p and q, MSE is calculated 50 times because of random variable and mean of 50 MSEs is taken as the MSE for these fixed p and q. Furthermore, MSEs are calculated for all combinations of p and q in the same way. After the program calculates MSE for $p = q = 1$ we have a matrix which includes 121 MSE for all combination of p and q. It searches for minimum MSE and the probabilities p and q which obtain this minimum MSE. However, as we me mentioned above since there exist random variables, we calculate these probabilites p and q 50 times and the means of these probabilities are chosen as optimum probabilites. After finding the optimal p and q, we again run the program to find the MSE for optimum p and q. (The code of Matlab is given in Appendix.) Thus, we have three different data set now:one is real stock price data, one is the data which is estimated using discretized GBM and the other is the data which is modeled using discretized version of modified GBM.

Section 5

Results

As we mentioned above, Mean Squared Errors (MSE) is one of the statistical method which gives us better fit model.We calculate MSE_1 for discretized GBM and MSE_2 for our discretized version of modified GBM. (The results are attached to the appendix.)

By using classical Geometric Brownian Motion process, we modified a new GBM model using different data partitions which are seperated two parts as positive and negative paths. Our modified GBM model is obtained by using the probabilities p and q. p is the probability which the stock prices will follow positive process if the return is positive and q is the probability which the stock prices will follow negative process if the return is negative. Figure 3, Figure 4 and Table 1 show us the histograms and statistics of p and q.

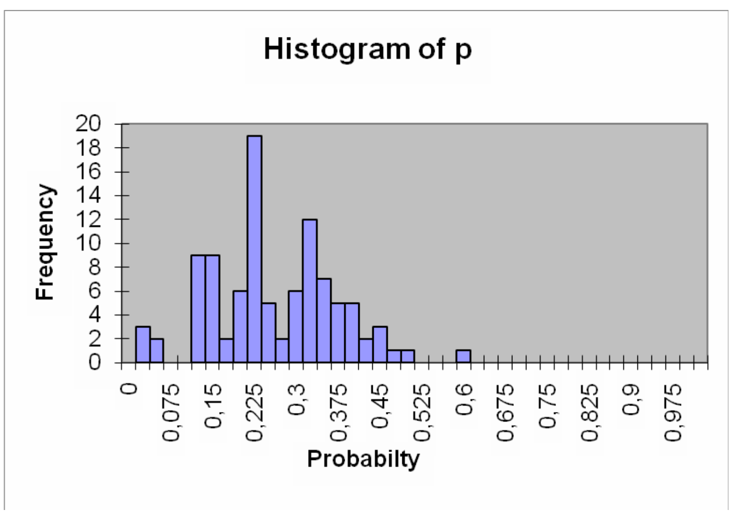

Figure 3

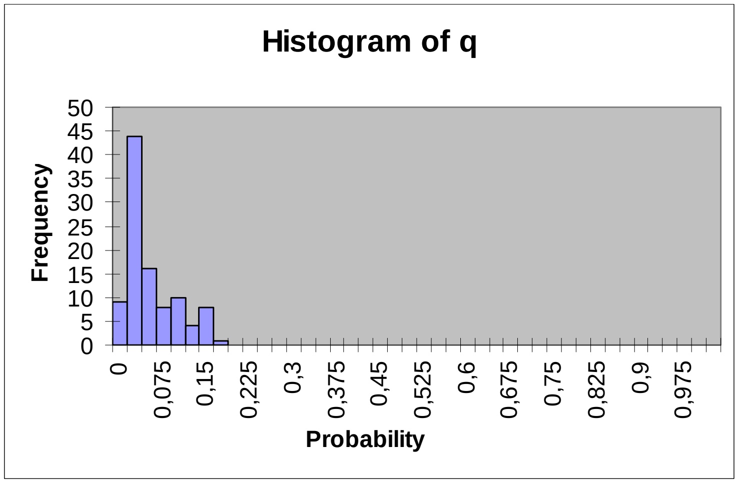

Figure 4

p		*q*	
Mean	0,24802	Mean	0,0413
Standard Error	0,011198286	Standard Error	0,004372307
Median	0,227	Median	0,021
Mode	0,308	Mode	0
Standard Deviation	0,111982861	Standard Deviation	0,043723073
Sample Variance	0,012540161	Sample Variance	0,001911707
Kurtosis	2,943610	Kurtosis	2,856139
Skewness	0,190989	Skewness	1,061910
Range	0,582	Range	0,156
Minimum	0,006	Minimum	0
Maximum	0,588	Maximum	0,156
Sum	24,802	Sum	4,13
Count	100	Count	100

Table 1: Statistics of p and q

The mean values of p and q, are 0.248 and 0.041, respectively. In addition to that, skewness of p and q are 0.191 and 1.062, respectively. It means that both of the probabilities have right skewed. In other words, they have relatively few high values. When we look at these values, we obviously realize that the probabilities p and q take low values. That means if the stock prices have positive returns today, they will follow negative process tomorrow and if the stock prices have negative returns, they follow positive process. Addionally, according to the values of probabilites it can be said that p which is the probability of following negative process during negative logreturn is lower than q which is the probability of following positive during the positive logreturns. As a result, we can say that positive return pursue negative process and negative return pursue positive process.

Section 7

Conclusions and Future Work

The aim of this study is to develop a discretized version of modified nonlinear GBM. As we mentioned above, our discretized version of modified GBM is formed of two different process as positive return path and negative return path. If logreturn of stock prices is positive at time t, then the process follows positive path with probability p or negative path with probability $1 - p$ at time $t + 1$. On the other hand, if logreturn of stock prices is negative at time t, then the process follows negative path with probability q or positive path with probability $1 - q$ at time $t + 1$. When we look at the probabilities p and q which minimize MSE, we clearly realize that stock prices follow the process of negative logreturn with higher probability at time $t + 1$ if the logreturn is positive at time t, vice versa. This may be attributable to the expectation of the investors which changes by the movements of stock prices. In other words, if the stock prices increase then the investors expect that the prices will decrease, begin to sell their stocks and the stock prices will reduce, vice versa.

So far, we compose a discrete version of nonlinear GBM which is designed to give better than GBM according to MSE. To develop this study, we will compare our model and another nonlinear GBM which is developed by a barrier strategy. It means that, we will try to show that our model gives better fit than this GBM model according to MSE. On the other hand, we may show that modified GBM gives us a better pricing results than GBM by using option pricing model. Hence, we require to develop this study to show that modified GBM gives better prediction than GBM for option price model. On the other hand, we can improve our modified GBM model by using stochastic volatility models.

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Appendix

i.Matlab Codes

```
x=xlsread('asset');
results=[];
t=length(x(:,1));
for row=1:100
N =50;
M=50;
y=[];
z=[];
h=[];
l=[];
k1=[];
k2=[];
f=[];
ms2=[];
mse2=[];
mmse2=[];
ms12=[];
mse12=[];
y=log(x(:,row));
ab=isnan(x(:,row));
to=sum(ab);
ko=t-to;
dif=(-y(1:t-1)+y(2:t))';
m = mean(dif(1:ko-1));
```

```
 for i=1:t-1
 k(i)=(dif(i)-m)^2;
 end
varyans = mean(k(1:ko-1));
nu = m+varyans/2;
dt=1;
z(1)=y(1);
z(2)=y(2);
for j=1:N
 e=normrnd(0,1,1,t);
 for i=3:ko
 z(i)=z(i-1)+(nu-varyans/2)*dt+sqrt(varyans)*sqrt(dt)*e(i);
 ms1(i)=(z(i)-y(i))^2;
 end
 mse1(j)=mean(ms1);
end
mmse1=mean(mse1);
m=1;
```

```
n=1;
for i=2:ko
 if y(i)>=y(i-1)
 h(m)=y(i)-y(i-1);
 m=m+1;
 else
 l(n)=y(i)-y(i-1);
 n=n+1;
 end
end
m1=mean(h);
for i=1:m-1
 k1(i)=(h(i)-m1)^2;
end
varyans1=mean(k1);
nu1=m1+varyans1/2;
m2=mean(l);
for i=1:n-1
 k2(i)=(l(i)-m2)^2;
end
varyans2=mean(k2);
nu2=m2+varyans2/2;
f(1)=y(1);
f(2)=y(2);
d=1;
for dr=1:M
for r=0:0.1:1
 for rr=0:0.1:1
```

```
for j=1:N
 u=rand(1,t);
 e1=normrnd(0,1,1,t);
 e2=normrnd(0,1,1,t);
 e3=normrnd(0,1,1,t);
 e4=normrnd(0,1,1,t);
 for i=2:ko-1
 if f(i)>f(i-1)
 if r>=u(i)
 f(i+1)=f(i)+(nu1-varyans1/2)*dt+sqrt(varyans1)*sqrt(dt)*e1(i);
 else
 f(i+1)=f(i)+(nu2-varyans2/2)*dt+sqrt(varyans2)*sqrt(dt)*e2(i);
 end
 else
 if rr>=u(i)
 f(i+1)=f(i)+(nu2-varyans2/2)*dt+sqrt(varyans2)*sqrt(dt)*e3(i);
 else
 f(i+1)=f(i)+(nu1-varyans1/2)*dt+sqrt(varyans1)*sqrt(dt)*e4(i);
```

```
 end
 end
 ms2(i)=(f(i)-y(i))^2;
 end
 mse2(j)=mean(ms2);
end
 mmse2(d)=mean(mse2);
 d=d+1;
 end
end
[mmse22,ii]=min(mmse2);
kt=floor(ii/11);
if ii-kt*11==0
 r1=(kt-1)*0.1;
 rr1=1;
else
 r1=kt*0.1;
 rr1=(ii-kt*11-1)*0.1;
end
prob(dr,1)=r1;
prob(dr,2)=rr1;
r1=0;
rr1=0;
d=1;
end
r12=mean(prob(:,1));
rr12=mean(prob(:,2));
for j=1:N
```

```
 u=rand(1,t);
 e1=normrnd(0,1,1,t);
 e2=normrnd(0,1,1,t);
 e3=normrnd(0,1,1,t);
 e4=normrnd(0,1,1,t);
 for i=2:ko-1
 if f(i)>f(i-1)
 if r12>=u(i)
 f(i+1)=f(i)+(nu1-varyans1/2)*dt+sqrt(varyans1)*sqrt(dt)*e1(i);
 else
 f(i+1)=f(i)+(nu2-varyans2/2)*dt+sqrt(varyans2)*sqrt(dt)*e2(i);
 end
 else
 if rr12>=u(i)
 f(i+1)=f(i)+(nu2-varyans2/2)*dt+sqrt(varyans2)*sqrt(dt)*e3(i);
 else
 f(i+1)=f(i)+(nu1-varyans1/2)*dt+sqrt(varyans1)*sqrt(dt)*e4(i);
 end
 end
 ms12(i)=(f(i)-y(i))^2;
 end
 mse12(j)=mean(ms12);
end
mmse12=mean(mse12);
results(row,1)=r12;
results(row,2)=rr12;
results(row,3)=mmse1;
results(row,4)=mmse12;
```

end

ii. Estimation Outputs

Stocks	p	q	mse1	mse2
adana	0,304	0,004	0,980	0,435
aefes	0,356	0,156	0,543	0,453
afyon	0,204	0,004	1,249	0,790
agyo	0,400	0,000	1,134	0,743
akbnk	0,202	0,002	1,016	0,631
akcns	0,204	0,004	1,013	0,478
akenr	0,210	0,010	0,716	0,327
akgrt	0,280	0,080	1,058	0,751
aksa	0,104	0,004	0,504	0,221
alark	0,210	0,010	0,536	0,532
albrk	0,030	0,030	0,709	0,086
alkim	0,208	0,008	0,889	0,384
anhyt	0,316	0,116	1,825	0,900
angsr	0,404	0,004	1,200	0,534
arclk	0,138	0,038	0,988	0,708
asels	0,182	0,068	1,050	0,639
asyab	0,150	0,050	0,767	0,383
aygaz	0,166	0,066	0,460	0,320
bagfs	0,340	0,140	0,821	0,667
banvt	0,224	0,024	0,596	0,577
bimas	0,246	0,046	0,385	0,250
bjkas	0,006	0,006	2,091	1,227
ccola	0,214	0,014	7,337	0,186
clebi	0,120	0,020	0,955	0,560
deva	0,308	0,008	1,566	1,046
dgzte	0,040	0,016	1,328	0,720
doas	0,130	0,030	0,884	0,389
dohol	0,360	0,060	1,198	0,685
dyhol	0,230	0,064	1,824	0,995
ecilc	0,400	0,000	0,939	0,625
eczyt	0,332	0,132	0,662	0,421
eggub	0,208	0,008	1,114	0,633
egser	0,212	0,012	2,134	0,842
enkai	0,266	0,076	9,681	6,302
eregli	0,300	0,000	1,002	0,518
fener	0,464	0,092	0,712	0,360
fenis	0,370	0,070	1,433	0,672
ffkrl	0,356	0,056	2,026	1,036
finbn	0,400	0,000	1,483	0,799
fortis	0,308	0,008	1,778	0,847
froto	0,308	0,108	0,857	0,631
garanti	0,334	0,134	1,054	0,745
golds	0,252	0,052	1,325	0,693
gsdho	0,212	0,012	1,220	0,554
gubrf	0,304	0,004	1,956	0,854
gyho	0,308	0,008	1,281	0,716
halkbnk	0,112	0,012	0,950	0,174
hurgz	0,198	0,098	1,037	1,068
iheva	0,216	0,016	1,942	0,988
ihlas	0,218	0,018	1,136	0,702
isctr	0,206	0,006	1,251	0,482

isfin	0,492	0,092	2,267	1,677
isgyo	0,300	0,000	1,216	0,716
izmdc	0,330	0,130	0,821	0,898
karsn	0,120	0,020	0,928	0,743
kchol	0,186	0,086	0,571	0,671
kipa	0,312	0,012	0,979	0,579
koza	0,216	0,016	1,841	0,964
krdmd	0,588	0,088	1,905	1,458
marti	0,312	0,084	1,379	0,993
nthol	0,402	0,002	1,295	1,078
nttur	0,302	0,020	1,302	0,964
otkar	0,334	0,134	0,812	0,483
pegyo	0,430	0,130	1,421	1,047
petk	0,202	0,004	0,731	0,492
petum	0,436	0,136	0,694	0,635
pnsut	0,296	0,096	1,500	0,714
prkte	0,142	0,010	1,285	0,739
ptofs	0,190	0,018	0,720	0,452
pysas	0,122	0,022	0,620	0,435
sahol	0,200	0,000	0,734	0,667
sasa	0,314	0,014	0,752	0,607
seles	0,016	0,016	0,205	0,148
sise	0,382	0,122	1,078	0,596
skbnk	0,308	0,008	1,767	1,054
sngyo	0,016	0,116	5,386	0,270
tatks	0,144	0,030	0,606	0,480
tavhl	0,118	0,018	0,687	0,188
tcell	0,202	0,002	0,635	0,281
tebank	0,436	0,136	1,658	0,929
tekfn	0,356	0,056	2,044	1,077
thyoa	0,164	0,036	0,870	0,342
tire	0,326	0,026	1,038	0,361
tkstl	0,124	0,024	0,638	0,194
toaso	0,200	0,000	1,278	0,631
tprs	0,246	0,046	3,130	2,235
trcas	0,300	0,000	0,659	0,405
trkcm	0,400	0,000	1,655	0,916
tskm	0,328	0,028	0,601	0,263
tspor	0,112	0,016	0,453	0,050
ttkm	0,204	0,010	0,609	0,503
ttrak	0,294	0,094	0,791	0,448
tupras	0,244	0,044	1,936	1,195
vakifbnk	0,108	0,008	0,641	0,286
vanet	0,238	0,038	0,989	0,503
vesbe	0,146	0,044	0,510	0,338
vestel	0,134	0,034	0,829	0,536
ykbnk	0,204	0,004	0,883	0,626
yksrg	0,126	0,026	1,431	0,508
zoren	0,130	0,030	0,740	0,354