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Universal size effect of concrete specimens and effect of notch depth

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ABSTRACT

The universal size effect law of concrete is a law that describes the dependence of nominal strength of specimens or structure on both its size and the crack (or notch) length, over the entire of interest, and exhibits the correct small and large size asymptotic properties as required. The main difficulty has been the transition of crack length from 0, in which case the size effect mode is Type 1, to deep cracks (or notches), in which case the size effect mode is Type 2 and fundamentally different from Type 1. The current study is based on recently obtained comprehensive fracture test data from three-point bending beams tested under identical conditions. In this test, the experimental program consisted of 80 three-point bend beams with 4 different depths 40, 93, 215 and 500mm, corresponding to a size range of 1:12.5. Five different relative notch lengths, a/D = 0, 0.02, 0.075, 0.15, 0.30 were cut into the beams. A total of 20 different geometries (family of beams) were tested. The present paper will use these data to analyze the effects of size, crack length. This paper presents a studying to improve the existing universal size effect law, named by Bazant, using the experimentally obtained beam strengths for various different specimen sizes and all notch depths. The updated universal size effect law is shown to fit the comprehensive data quite well.

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1. Introduction

The proceeding conference articles and paper (Şener at al., 2014a; Çağlar and Şener, 2015; Çağlar and Şener, 2016; Şener and Şener, 2016) presented an introduction to the problem and reported comprehensive test data for fracture of concrete specimens. The experimental program, also described in (Şener et al., 2014b), consisted of 80 three-point bend beams with 4 different depths 40, 93, 215 and 500mm, corresponding to a size range of 1:12.5. Five different relative notch lengths, a/D = 0, 0.02, 0.075, 0.15, 0.30 were cut into the beams. A total of 20 different geometries (family of beams) were tested. The present paper will use these data to analyze the effects of size, crack length. A special case of this law is a formula for the effect of notch or crack depth at fixed specimen size, which overcomes the limitations of a recently proposed empirical formula by Duan et al. (2003, 2006).

The Scientific and Technological Research Council of Turkey (TUBITAK) provided funding to carry out comprehensive fracture tests of beam specimens made from the almost the same age and same concrete mix to investigate the influence of size and notch length on specimen strength.

2. Reviews of Size Effect and Crack Length Effect

The nominal strength of geometrically similar structures, defined with Eq. (1) is

$$\sigma_N = c_N \frac{P_u}{bD}, \quad (1)$$

independent of structure size D (P = maximum load; b = structure width; and c_N = dimensionless constant chosen for convenience). Size effect is defined as any dependence

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of σ_N on D, which is a phenomenon typical in fracture or damage mechanics.

According to linear elastic fracture mechanics (LEFM) theory, which applies to homogeneous perfectly brittle materials, and for geometrically similar structures with similar cracks, $\sigma_N \propto D^{-1/2}$, which is the strongest possible size effect. For quasi-brittle materials such as concrete, one can distinguish two simple types of size effect as shown in Eq. (2).

$$\sigma_N = \frac{Bf_t}{\sqrt{1+D/D_0}}. \tag{2}$$

Here B and the transitional structure size D_0 are empirical parameters to be identified by data fitting and f_t is tensile strength of concrete introduced for convenience. Eq. (2) was derived (Bazant, 1984) by simple energy release analysis and later by several different approaches such as by asymptotic matching based of the asymptotic power scaling laws for very large and very small D (Bazant and Planas, 1998). In the standard size effect plot of log σ_N versus log D, Eq. (2) gives a smooth transition from a horizontal asymptote to an inclined asymptote of slope -1/2 (Fig. 1).

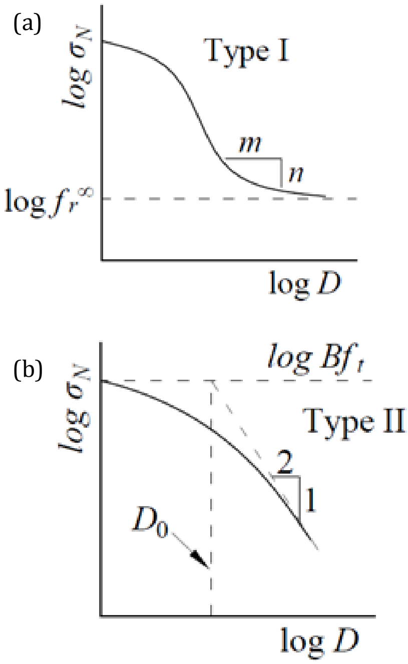

Fig. 1. Dependence of σ_N on structure size D of beams with (a) no notched and (b) deep notch.

In Eq. (2),

$$Bf_t = \sqrt{\frac{E'G_f}{g_0' c_f}}, \quad D_0 = \frac{c_f g_0'}{g_0}, \tag{3}$$

where $g_0 = g(\alpha_0)$; $g_0' = g'(\alpha_0)$; $\alpha = a/D$ = relative crack length; $\alpha_0 = a_0/D$ = initial value of α; $g(\alpha) = k^2(\alpha)$ = dimensionless energy release rate function $g(\alpha)$ of LEFM; $k(\alpha) = b\sqrt{(DK_I/P)}$ where K_I = stress intensity factor, P = load; $g'(\alpha) = dg(\alpha)/d\alpha$, $E' = E$ = Young's modulus for plane stress and $E' = E/(1-\nu^2)$ for plane strain (where ν = Poisson ratio), G_f=initial fracture energy = area under the initial tangent of the cohesive softening stress-separation curve; c_f = characteristic length, which represents about a half of the Fracture Process Zone (FPZ) length. Eq. (2) may be rewritten as shown in Eq. (4).

$$\sigma_N = \sqrt{\frac{E'G_f}{g_0 D + g_0' c_f}}. \tag{4}$$

Because function $g(\alpha)$ or $k(\alpha)$ embodies information on the effects of crack length and structure geometry, Eq. (4) is actually a size effect law for Type 2 failures.

The Type 1 size effect, σ_N approaches, for large D, a constant value (a horizontal asymptote in the size effect plot), since the Weibull statistical size effect (Weibull, 1939) is unimportant. For three point bend beams, it is indeed unimportant. Because the zone of high stresses is rather concentrated, even do not exist along a notch. This prevents the critical crack from forming at widely different locations of different random local strength (for the same reason, the statistical size effect is negligible in Type 2 failures also).

The large size asymptote for Type 1 size effect is, in the log-log plot, a downward inclined straight line of a slope $-n/m$, which is much milder than the slope of -1/2 for LEFM (Weibull, 1939) (Fig.1); here m = Weibull modulus and n = number of spatial dimensions of fracture scaling (n = 2 for the present tests). The small size asymptote is also a horizontal line and, for medium sizes, the size effect is a transition between these two asymptotes. In absence of the statistical size effect, Eq. (5) was used by Hoover and Bazant (2014).

$$\sigma_N = f_r^{\infty}\left(1 + \frac{rD_b}{D+l_p}\right)^{1/r}. \tag{5}$$

Here f_r^{∞}, D_b, l_p, and r are empirical constants to be determined from tests; f_r^{∞}=nominal strength for very large structures, assuming no statistical size effect (in the special case of very large beams, f_r^{∞} represents the flexural strength, also called the modulus of rupture); and D_b = depth of the boundary layer of cracking (roughly equal to the FPZ size). In all previous works, D = same characteristic structure size as used for the Type 2 size effect (Eq. (4)). Furthermore, l_p = material characteristic length, which is related to the maximum aggregate size d_a. If the structure is larger than $10l_p$, one can set $l_p \approx 0$, which corresponds to the original formulation of the Type I law.

It was further shown that the Type 1 and 2 Size Effect Laws (SELs) satisfy the large-size and small size asymptotic properties of the cohesive crack model applied to Type 1 and 2 failures. Furthermore, it was experimentally confirmed that, within the range of inevitable experimental scatter, the SEL of Type 2 gives about the same values of fracture energy G_f when applied to notched fracture specimens (e.g., compact compression test (Abusiaf et al., 1996; Barr et al., 1998), torsional test (Abusiaf et al., 1997)).

3. Application of Universal Size Effect Law by Fracture Tests

To calibrate the deterministic Universal Size Effect Law (USEL), the mean of data was computed separately for each family of identical specimens from comprehensive fracture tests (Şener et al., 2014a; 2014b), Çağlar and Şener 2015). The surface of the optimized USEL is plotted in Fig. 2. In this Fig. 2 size effect curves were given for only $\alpha = 0$, 0.15 and 0.3. Transition from these curves for calibrating USEL is just used with smooth curves. The studies on these transition curves are still on going.

In particular, the fracture parameters G_f and c_f should not be influenced by the data for beams with no notches (Type I data) or shallow notches and f_r^{∞}, D_b, l_p and r should not be influenced by the data for deep notches. Therefore, these parameters were determined first by separate fitting of specimens with deep notches ($\alpha = 0.30$ or 0.15) and specimens with shallow or no notches ($\alpha = 0$). Only the nonstatistical USEL (Bazant and Yu, 2009) in Eq. (6) was considered. Nonlinear fitting of the Type I, SEL (Eq. (5)) to the notchless ($\alpha = 0$) beams gave (Şener et al., 2014a, b) values in Eq. (6) with coefficient of variation of fit 9.4%.

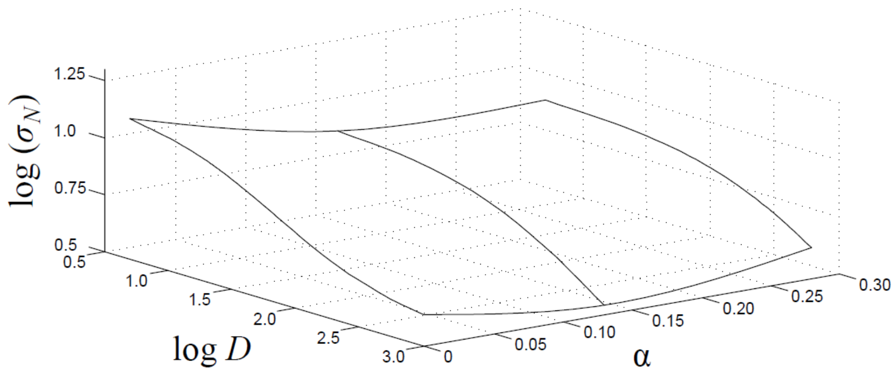

Fig. 2. Entire Universal Size Effect law surface.

$$D_b = 90\,mm,\; l_p = 50\,mm,\; f_r^{\infty} = 4\,MPa,\; r = 0.52\,. \quad (6)$$

These values are different from than the studies by Hoover and Bazant's (2014) D_b=73.2 mm, l_p=126.6 mm, f_r^{∞}=5.27 MPa. The difference between some of the parameters was in the order of two for especially l_p value. The size range 1:12.5 was large enough to identify all the fracture parameters in Eq. (5). The USEL can be drawn for a fixed α, which gives a size effect plot of log(σ_N) versus log D (Fig. 2).

In Fig. 3, this plot is created and compared with the data from Şener et al. (2014a, 2014b). The results obtained from the tests for Type II size effect (Eq. 2) was used for deep (α=0.3) and big (α=0.15) notches are shown at the Figs. 3(a, b). For the crack initiation specimens were fitted using Eq. (5), and the resulting constants were calculated. The calculated constants are given in Table 1 for the shallow notch (Fig. 3(d)) and notchless beams (Fig. 3(e)). Type I parameters, which are presented in parenthesis in Table 1 were compared with Bazant's (Hoover and Bazant, 2014) results. The parameters obtained from the results of shallow notch specimens were not compared with Bazant's results, because of insufficient data in their work for α=0.02. But Type I size effect parameters for notchless and shallow notch specimens obtained from this study were consistent between the two test programs.

In Fig. 3, for α=0.3 and 0.15, Type II (Figs. 3(a, b)) size effect was used, for unnotched specimen α=0, Type I (Fig. 3(e, f)) size effect was used. For the medium size notched (α=0.075) beams, the failure stress was in between the Type I and Type II curves, so these curves are not shown in the figure (Fig. 3(c)).

Table 1. Type I size effect coefficients.

α	D_b (mm)	l_p (mm)	f_r^{∞} (MPa)	r
0	90	50	4	0.52
(0)	(73.2)	(126.6)	(5.27)	(0.52)*
0.02	110	66	3.2	0.52

*Parameters inside parenthesis are Bazant's (Hoover and Bazant, 2014) values.

4. Comparison with Duan-Hu's Boundary Effect Model

The analysis of the test results were performed using Bazant's Type II size effect formulas and Type I. There are many widely accepted and practical size effect evaluation approaches such as; multifractal scaling law of Carpinteri (Carpinteri et al., 1995), asymptotic analysis of size effect of Karihaloo (Karihaloo et al., 2003), and boundary effect model of Hu and Duan (Duan et al., 2003, 2006). The size effect model of Hu and Duan is a boundary effect model, which was recently developed, by scaling of quasi-brittle size effect on strength of finite sized specimens. The test results were also analyzed using Hu and Duan's approach for comparison with Type I size effect of specimens with α=0. The nominal strength (σ_N) formula of Hu and Duan which accounts for size effect in concrete is given in Eq. (7) for un-notched specimens.

$$\sigma_N = \sigma_o (1 + B_1 D)^{-0.5}\,, \quad (7)$$

where σ_0 is the maximum tensile stress in the ligament based on a linear stress distribution over the ligament based on three-point bend specimens and B_1 is a constant. σ_0 and B_1 can be obtaining from linear regression analysis. Eq. (7) is mathematically similar to the Type II size effect formula given in Eq. (2).

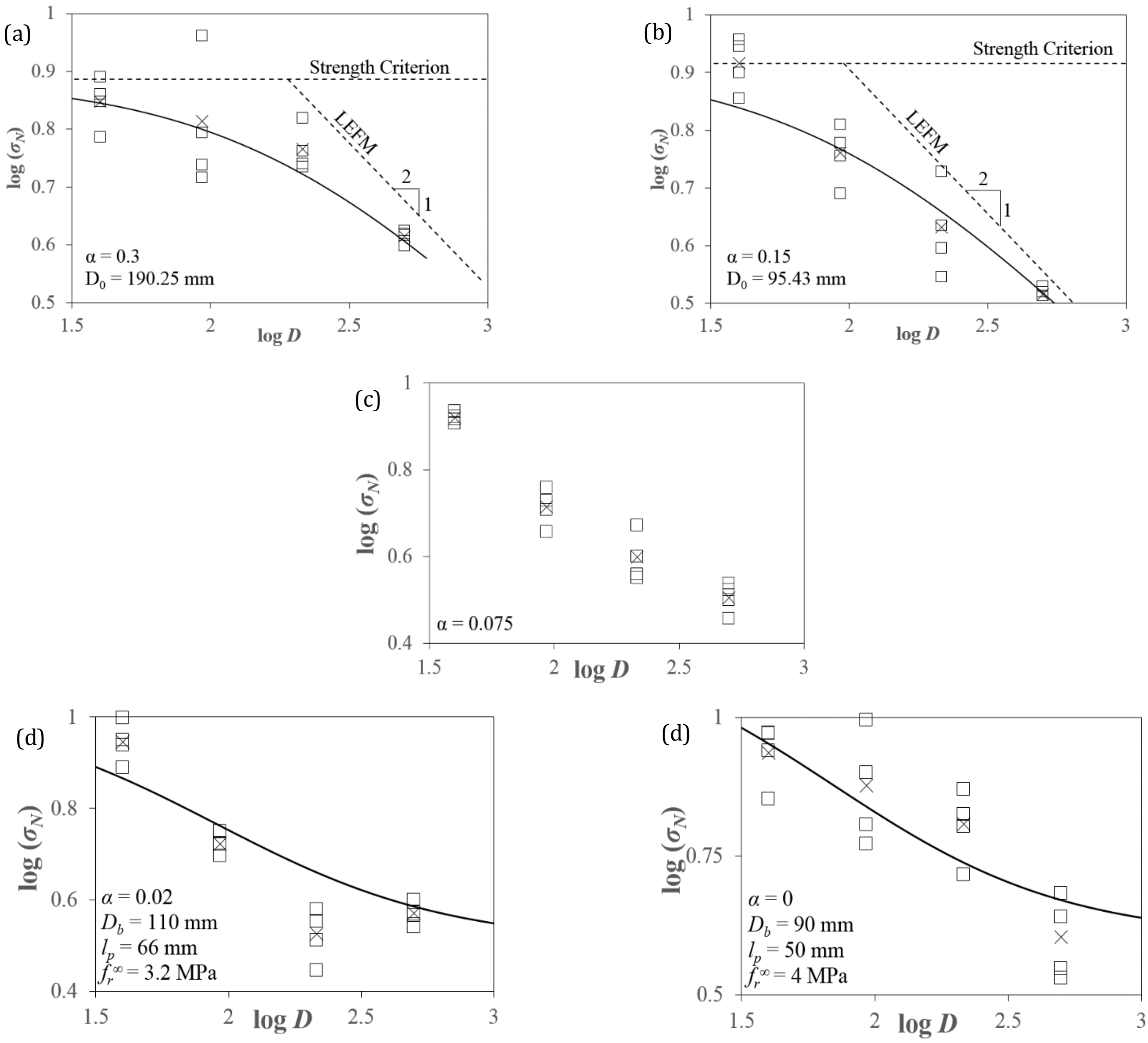

Fig. 3. Effect of structure size on the nominal strength of the data from Şener et al. (2014a).

The comparison of size effect plots of Type I and Duan's model are given in Fig. 4 for α=0. The plots indicate that Type I size effect and Duan's boundary effect model differs significantly for members' smaller depth. In the Type I (solid line) Eq. (2) the parameters were taken from Table 1 for α=0, for Duan's (broken line) Eq. (7) σ_0= 11.62 MPa, B_1= 0.0135 values were used.

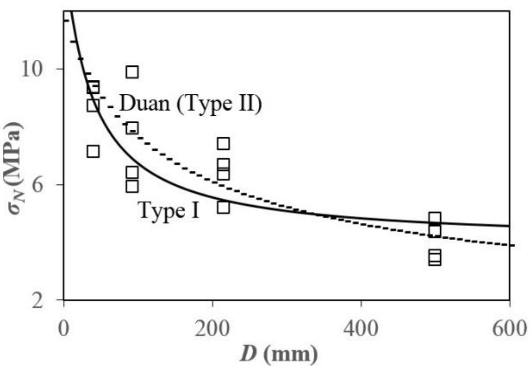

Fig. 4. Size effect plots of test results overlaid with Type I and Duan's size effect model for α=0.

5. Conclusions

- The Type 2 size effect in specimens with deep notches or cracks does not give a correct transition to of Type 1 in specimens with no notch or crack.
- The size effect data from deeply notched specimens (α=0.3 and 0.15), and parameters f_r^{∞}, D_b, l_p, and r were determined separately by fitting only the size effect data for unnotched specimens (α=0).
- USEL fits the measured nominal strength quite well.
- Both Type I and Type II size effect were observed in this study and confirmed the need to be account for size effect in design codes.
- The comparison of Type I and Duan's boundary effect formulas (Type II) exhibit difference for members with small depth for α=0.

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