ON REFINEMENTS OF APPROVAL VOTING

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Onaylı Seçimin Rafineleştirilmesi Üzerine

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Abstract

Approval voting is the voting procedure that selects the candidates who get the most votes in a society where voters are allowed to approve of as many candidates as they wish. In this study, we focus on approval voting as a social choice correspondence which selects the alternatives that at given preference profile there exists admissible and sincere approval profile such that the voting procedure selects. We study Maskin monotonic refinements of approval voting in order to find its minimal refinement. We construct a social choice correspondence based on the number of approvals; and we show the properties of this refinement.

Özet

Onaylı seçim oy kullananların istediği sayıda adaya oy vermesine izin verilen bir toplulukta en fazla oy alan adayın galip geldiği seçim prosedürüdür. Bu çalışmada Onaylı seçim, bu prosedür tarafından verilen tercih profiline göre yapılan olası kabul edilebilir ve samimi oylamalarda kazanabilecek adayları seçen küme değerli sosyal seçim kuralı olarak ele alınmıştır. Onaylı seçimin minimal rafineleştirmesini bulmak için Maskin monoton rafineleştirmeleri üzerine çalıştık, ve kullanılan oyların sayısına bağlı olan bir küme değerli sosyal seçim kuralı oluşturarak; bu rafineleştirmenin özelliklerini inceledik.

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Contents

Chapter 1

Introduction

Approval voting is a voting procedure that selects the candidate which gets most votes while voters are allowed to vote for as many candidates as they want. Approval voting or voting systems within the spirit of Approval voting were used through the history. For example, Lines (1986) shows the role of Approval voting in the election of Doge of Venice for more than five hundred years. Although been practically used for such a long time, Approval voting was formally defined and its properties were analysed for the first time by Brams and Fishburn (1978). Five years after their article, Brams and Fishburn published a seminal book about Approval voting (Brams and Fishburn 1983). Among their book, Brams and Fishburn published several papers about Approval voting. For example, Brams and Fishburn (1981) study the efficacy and equity concepts for Approval voting; Brams and Fishburn (1992) examine elections of four scientific and engineering societies that used Approval voting between 1987 and 1988; and Brams and Fishburn (2005) discuss Approval votings adoption by societies.

Brams and Fishburn were not the only academicians who were interested in Approval voting. For example; Weber (1995) published his thoughts about Approval voting; Alos-Ferrer (2006) discusses Approval voting as ballot aggregation function and he states that it is the only one that satisfies faithfulness, consistency and cancellation; Laslier and Straeten (2008) study the experiment about Approval voting which took place during the 2002 French presidential election; and Laslier (2009) examines behaviours of voters in Approval voting when the society is very large. Lately Laslier and Sanver published a book entitled "Handbook on Approval Voting", which summarizes the literature on Approval voting (Laslier and Sanver 2010).

Approval voting is a voting procedure which is defined on sets of approved alternatives. To extend spirit of Approval voting into usual social choice framework; we take into account voters preference relations and the information of how many candidates they will approve. Then we define approval voting as a social choice correspondence, where it selects the set of alternatives that can be selected by voting procedure in some admissible and sincere approval profile. Approval voting, which is defined on preference profile, has a very nice property; it satisfies Maskin monotonicity that is necessary condition for a social choice correspondence to be Nash implementable (Maskin 1999). Despite this nice property, Approval voting has a disadvantage; it selects a large set of alternatives. So, we want to refine Approval voting, while we preserve Maskin monotonicity. To refine Approval voting without losing Maskin monotonicity, we construct a social choice correspondence, the refined approval voting, depends on the number of approvals, and show this correspondence is a Maskin monotonic refinement of Approval voting .

This thesis is organized as follows; Chapter 2 gives basic definitions and notations that will be used in the following chapters. Chapter 3 mentions

Maskin monotonicity, and some properties of Maskin monotonic social choice correspondences. Finally, Chapter 4 gives formal definition of Approval voting, and mentions our findings on refinements of Approval voting.

Chapter 2

Preliminaries

In this chapter, we give basic definitions and notations that are used in this thesis.

The set $N = \{1, ..., n\}$ with $n \geq 2$ denotes the society which confronts a set of alternatives $A = \{a_1, ..., a_m\}$ with $m \geq 2$.

Definition 2.1 *Given any two sets S and T, their Cartesian product is the set $S \times T = \{(s, t) | s \in S$ and $t \in T\}$. An element of a Cartesian product is called an ordered pair.*

Definition 2.2 *Given a set S, a binary relation R on S is a subset of the Cartesian product $S \times S$. An ordered pair $(x, y) \in R$ is interpreted as x is related with y.*

Notation 2.1 *Given a set S, and for any $a, b \in S$; we will simplify the notation $(a, b) \in R$ by aRb.*

Definition 2.3 *Given a set S, a binary relation R on S is complete if and only if $\forall x, y \in S$, we have xRy or yRx.*

Definition 2.4 *Given a set S, a binary relation R on S is transitive if and only if $\forall x, y, z \in S$, we have xRy and $yRz \Rightarrow xRz$.*

Definition 2.5 *Given a set* S*, a binary relation* R *on* S *is antisymmetric if and only if* $\forall x, y \in S$*, we have* xRy *and* $yRx \Rightarrow x = y$*.*

Definition 2.6 *Strict preference relation of an individual* $i \in N$ *over the alternative set* A *is a complete, transitive and antisymmetric binary relation* $P_i \in \Pi$*, where* Π *is the set of complete, transitive and antisymmetric binary relations over* A*. Then* $P = (P_1, ..., P_n) \in \Pi^N$ *stands for the preference profile of society.*

Definition 2.7 *Given a strict preference relation* P_i *over* A*; the lower contour set of alternative* $x \in A$ *with respect to* P_i *is the set*
$L(x; P_i) = \{y \in A : xP_iy\}$*.*

Definition 2.8 *The rank of an alternative* x *in a preference relation* P_i *is* $r(x; P_i) = m - \#L(x; P_i) + 1$*.*

Definition 2.9 *A social choice correspondence (SCC)* $F : \Pi^N \longrightarrow 2^A \backslash \{\varnothing\}$ *is a mapping from* $\Pi^{\mathbf{N}}$ *into* A*, that selects a non-empty subset of* A *for each possible preference profile of the society.*

Definition 2.10 *Given two SCCs* $F, G : \Pi^N \longrightarrow 2^A \backslash \{\varnothing\}$*; the union of* F *and* G *is a SCC* $F \cup G : \Pi^N \longrightarrow 2^A \backslash \{\varnothing\}$ *such that* $F \cup G(P) = F(P) \cup G(P)$ $\forall P \in \Pi^N$*.*

Definition 2.11 *Given two SCCs* $F, G : \Pi^N \longrightarrow 2^A \backslash \{\varnothing\}$*; the intersection of* F *and* G *is a mapping* $F \cap G : \Pi^N \longrightarrow 2^A$ *such that* $F \cap G(P) = F(P) \cap G(P)$ $\forall P \in \Pi^N$*.*

Remark 2.1 *Since* $F \cap G(P)$ *can be empty for some* $P \in \Pi^N$*, the intersection of two SCCs need not to be a social choice correspondence.*

Definition 2.12 *For any two n-tubles* $a = (a_1, ..., a_n) \in \mathbb{Z}^n$ *and* $b = (b_1, ..., b_n) \in \mathbb{Z}^n$*;* $a < b$ *if and only if* $a_i \leq b_i$ *for* $i \in \{1, ..., n\}$ *and* $a_j < b_j$ *for at least one* $j \in \{1, ..., n\}$.

Definition 2.13 *The ceiling function* $\lceil x \rceil = \min \{n \in \mathbb{Z} \mid n \geq x\}$ *maps a real number to smallest integer not less than* x.

Chapter 3

Maskin Monotonicity

Maskin monotonicity requires social choice correspondences to satisfy this condition; if a social choice correspondences selects an alternative at a profile; then this alternative must be selected in any other profile where for each agent the lower contour set of this alternative does not shrink. Maskin monotonicity is an intuitive condition; but also a very strong condition. Many well-known social choice correspondences do not satisfy this condition. For example, if indiffirences are not allowed in preference relations no scoring rule (Erdem and Sanver 2005) are Maskin monotonic.

We now give the formal definition of Maskin monotonicity.

Definition 3.1 *For any $x \in A$ and $P, P' \in \Pi^N$ with $L(x; P_i) \subseteq L(x; P'_i)$ $\forall i \in N$, we say that P' is an improvement for x with respect to P.*

Definition 3.2 *A SCC $F : \Pi^N \to 2^A \backslash \{\emptyset\}$ is called Maskin monotonic if and only if it satisfies the following condition for all $P \in \Pi^N$ and for all $x \in F(P)$; if $P' \in \Pi^N$ is an improvement for x with respect to P, then $x \in F(P')$.*

We restate a propositon that was stated and proved by Maskin (1985).

Proposition 3.1 *Given any two SCC's F, G with $F \cap G : \Pi^N \to 2^A \backslash \{\emptyset\}$; if F and G are Maskin monotonic then $F \cap G$ is Maskin monotonic.*

Proof. Take any $x \in A$ and any two profiles $P, P' \in \Pi^N$ such that P' is an improvement for $x \in A$ with respect to P. Let $x \in F \cap G(P)$, implies $x \in F(P)$ and $x \in G(P)$. Since F and G are Maskin monotonic, we have $x \in F(P')$ and $x \in G(P')$; implying $x \in F \cap G(P')$. This proves $F \cap G$ is Maskin monotonic. ■

3.1 Maskin Monotonic Refinements

Definition 3.3 *Given any two SCC's $F, G : \Pi^N \longrightarrow 2^A \backslash \{\varnothing\}$; G is called Maskin monotonic refinement of F if and only if $\varnothing \neq G(P) \subseteq F(P)\ \forall P \in \Pi^N$ and $G(P) \subset F(P)$ for some $P \in \Pi^N$, and G is Maskin monotonic.*

It's reasonable that usually minimal Maskin monotonic extension or maximal Maskin monotonic refinement of a non-monotonic social choice correspondence is studied to reach a Maskin monotonic correspondence with adding or subtraction as few as possible. For example, Erdem and Sanver (2005) presents minimal Maskin monotonic extension of scoring rules. But in our study we have a Maskin monotonic social choice correspondence, which selects a large set of alternatives. So to refine this correspondence as much as it is possible without dropping the Maskin monotonicity condition, we define minimal Maskin monotonic refinement of a social choice correspondence.

Definition 3.4 *G is called a minimal Maskin monotonic refinement of a SCC F, if G is a Maskin monotonic refinement of F and $\nexists G'$ such that G' is a Maskin monotonic refinement of G.*

Chapter 4

Approval Voting

Approval voting is a voting procedure that works with the idea of individuals vote for as many alternatives as they want. So this procedure makes individuals split the alternative set into two disjoint sets; which one set contains approved alternatives and the other contains the rest of the alternatives.

Definition 4.1 *Approval set* S_i, $\emptyset \neq S_i \subseteq A$, *is the set of alternatives that are approved by individual* i. *Then n-bundle* $S = (S_1, ..., S_n)$ *will be the approval profile of society.*

Definition 4.2 *Given any preference relation* $P_i \in \Pi$, *the set* S_i *is admissible with respect to* P_i *if and only if* $r(x, P_i) = 1 \Rightarrow x \in S_i$ *and* $r(x, P_i) = m \Rightarrow x \notin S_i$. *Given any preference profile* $P = (P_1, ..., P_n)$, *the n-tuple* S *is admissible with respect to preference profile* P *if* S_i *is admissible with respect to* P_i *for all* $i \in N$.

Definition 4.3 *Given any preference relation* $P_i \in \Pi$, *the set* S_i *is sincere with respect to* P_i *if and only if* $x \in S_i$ *and* $yP_ix \Rightarrow y \in S_i$. *Given any preference profile* $P = (P_1, ..., P_n)$, *the n-tuple* S *is sincere with respect to preference profile* P *if* S_i *is sincere with respect to* P_i *for all* $i \in N$.

Definition 4.4 *For any* $x \in A$, *and approval profile* S; *the number* $n(x; S) = \#\{i \in N : x \in S_i\}$ *is the number of individuals approving* x.

Definition 4.5 *Given an approval profile S, the voting procedure version of approval voting is* $\alpha(S) = \{x \in A : n(x; S) \geq n(y; S) \ \forall y \in A\}$.

Definition 4.6 *Approval voting* $F_\alpha : \Pi^N \to 2^A \backslash \{\emptyset\}$ *is a social choice correspondence with* $F_\alpha(P) = \{x \in A : x \in \alpha(S)$ *for some S which is sincere with respect to P*$\}$.

Brams and Sanver defined a critical approval profile for every alternative at any preference profile, such that to be selected by approval voting prodecure at its critical approval profile is a necessary and sufficient condition for an alternative to be an approval voting outcome (Brams and Sanver 2005). Now we properly define the critical approval profile and state the condititon as lemma. The proof of the lemma can be found in Brams and Sanver's paper.

Definition 4.7 *An approval profile* $S = (S_1, ..., S_n)$ *is critical approval profile for alternative x at preference profile P, if*
$S_i = \{y \in A : yP_ix\}$ *for all* $i \in N$ *with* $r(x, P_i) \neq m$ *and*
$S_i = \{y \in A : r(y, P_i = 1\}$ *for all* $i \in N$ *with* $r(x, P_i) = m$.

Lemma 4.1 *For any* $P \in \Pi^N$, $a \in F_\alpha(P)$ *if and only if* $a \in \alpha(S)$ *where S is the critical approval profile for a at P.*

Proposition 4.1 F_α *is Maskin monotonic.*

Proof. Take any $a \in A$ and any two preference profiles $P, P' \in \Pi^N$ with $a \in F_\alpha(P)$ and P' is an improvement for a with respect to P. By Lemma 4.1, $a \in F_\alpha(P)$ implies there exists critical approval profile S at P such that $a \in \alpha(S)$. So $n(a;S) \geq n(b;S)$ $\forall b \in A \backslash \{a\}$. Now let S' be the critical approval profile for a at P'. Since P' is an improvement for a with respect to P, we have $n(a;S') \geq n(a;S)$, and $n(b;S) \geq n(b;S')$ $\forall b \in A \backslash \{a\}$. So $n(a;S') \geq n(b;S')$ $\forall b \in A \backslash \{a\}$ implying $a \in F_\alpha(P')$. ■

Definition 4.8 *The approval index* $\mu = (\mu_1, ..., \mu_n) \in \{1, ..., m-1\}^N$ *is a n-tuble , where* μ_i *is the number of alternatives individual* i *approves.*

Remark 4.1 *Every preference profile* P *and approval index* μ *induces an approval profile* $S(\mu, P)$ *such that* $S_i(\mu_i, P_i) = \{x \in A : r(x, P_i) \leq \mu_i\}$ $\forall i \in N$.

Definition 4.9 *Given preference profile* P*, approval index* μ*, and* $c \in \{1, ..., m\}$*; the (*μ*,c)-approval voting* $F^c_\mu(P) = \{x \in A : n(x, S(\mu, P)) \geq c\}$ *is a mapping which picks alternatives that is approved by at least c individuals.*

Definition 4.10 *Given approval index* μ*, the critical score is the number* $\gamma(\mu) = \min_{P \in \Pi^N} \max_{x \in A} n(x, S(\mu, P))$.

Remark 4.2 *Given approval index* μ*,* $\gamma(\mu) = \lceil \frac{\sum_{i \in N} \mu_i}{m} \rceil$.

Proposition 4.2 *For any approval index* μ*, (*μ*,c)-approval voting rule is a social choice correspondence if and only if* $c \leq \gamma(\mu)$.

Proof. 'If' Take any μ and $c \leq \gamma(\mu)$. From the definition of $\gamma(\mu)$, we know $\forall P \in \Pi^N$ $\exists y \in A$ such that $n(y, S(\mu, P)) = \max_{x \in A} n(x, S(\mu, P)) \geq \gamma(\mu) \geq c$, implying $y \in F_\mu^c(P)$. So we get $F_\mu^c(P) \neq \emptyset$ $\forall P \in \Pi^N$, which shows (μ,c)-approval voting rule is a social choice correspondence.

'Only if' Take any μ, and let (μ,c)-approval voting rule $F_\mu^c(P)$ be a SCC. Assume for the contrary, $c > \gamma(\mu)$. From the definition of $\gamma(\mu)$, we know $\exists P \in \Pi^N$ such that $n(x, S(\mu, P)) \leq \gamma(\mu)$ $\forall x \in A$. But $n(x, S(\mu, P)) \leq \gamma(\mu) < c$ $\forall x \in A$ gives $F_\mu^c(P) = \emptyset$. which contradicts with the definition of SCC. ■

Remark 4.3 *For given μ, we have the relation $c < c' \Rightarrow F_\mu^c(P) \supseteq F_\mu^{c'}(P)$, that is derived from the definition of (μ,c)-approval voting . Our aim is to find the minimal Maskin monotonic refinement of approval voting, so we will use the case $c = \gamma(\mu)$, where $F_\mu^{\gamma(\mu)}(P) = \{x \in A : n(x, S(\mu, P)) \geq \gamma(\mu)\}$ is the finest (μ,c)-approval voting rule.*

Proposition 4.3 *Given approval index μ, the SCC $F_\mu^{\gamma(\mu)}(P)$ is Maskin monotonic.*

Proof. Take any μ, an alternative $x \in A$ and any two preference profiles $P, P' \in \Pi^N$ with $x \in F_\mu^{\gamma(\mu)}(P)$ and P' is an improvement for x with respect to P. Since $x \in F_\mu^{\gamma(\mu)}(P)$, we have $n(x, S(\mu, P)) \geq \gamma(\mu)$. Also P' is an improvement for x with respect to P gives us $n(x, S(\mu, P')) \geq n(x, S(\mu, P))$. When we combine these two equations, we get

$n(x, S(\mu, P')) \geq n(x, S(\mu, P)) \geq \gamma(\mu)$, which implies $x \in F_{\mu}^{\gamma(\mu)}(P')$. Hence $F_{\mu}^{\gamma(\mu)}(P)$ is Maskin monotonic. ■

Proposition 4.4 *Given preference profile $P \in \Pi^N$ and two approval indices μ and μ' with $\mu \neq \mu'$, having a relation between two indices (e.g. $\mu < \mu'$) does not give a relation between two finest (μ,c)-approval voting rules $F_{\mu}^{\gamma(\mu)}(P)$ and $F_{\mu'}^{\gamma(\mu')}(P)$ derived from them.*

Proof. Take $n = 9$, $A = \{a, b, c, d\}$, $\mu = (1, 1, 1, 1, 1, 1, 1, 1, 1)$, $\mu' = (2, 2, 2, 2, 2, 2, 2, 2, 2)$, and preference profile P as follows;

$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$	$\mathbf{P_4}$	$\mathbf{P_5}$	$\mathbf{P_6}$	$\mathbf{P_7}$	$\mathbf{P_8}$	$\mathbf{P_9}$
a	a	a	a	b	b	b	b	b
c	c	c	c	c	c	c	c	c
|	|	|	|	|	|	|	|	|
|	|	|	|	|	|	|	|	|

Here we have $\gamma(\mu) = \lceil \frac{\sum_{i \in N} \mu_i}{m} \rceil = \lceil \frac{9}{4} \rceil = 3$, so $F_{\mu}^{\gamma(\mu)}(P) = \{a, b\}$, and $\gamma(\mu') = \lceil \frac{\sum_{i \in N} \mu'_i}{m} \rceil = \lceil \frac{18}{4} \rceil = 5$ so $F_{\mu'}^{\gamma(\mu')}(P) = \{c\}$. So we have $\mu < \mu'$; but $F_{\mu}^{\gamma(\mu)}(P)$ and $F_{\mu'}^{\gamma(\mu')}(P)$ are two disjoint sets. ■

Finest (μ,c)-approval voting rule for given μ, is a good candidate to be a Maskin monotonic refinement of approval voting. But it may contain alternatives that are not in approval voting. We show this by the following example.

Example 1 *Take $n = 3$, $A = \{a, b, c\}$, $\mu' = (1, 2, 2)$ and preference profile P as follows;*

$\mathbf{P_1}$	$\mathbf{P_2}$	$\mathbf{P_3}$
c	c	c
$-$	a	a
b	$-$	$-$
a	b	b

In this example $\gamma(\mu') = \lceil \frac{\sum_{i\in N}\mu'_i}{m} \rceil = \lceil \frac{5}{3} \rceil = 2$, *so* $F_{\mu'}^{\gamma(\mu')}(P) = \{a, c\}$. *But* $a \notin F_\alpha(P)$, *because* $\forall \mu$ *we have* $n(c, S(\mu, P)) = 3$ *and* $n(a, S(\mu, P)) \leq 2$, *so* $n(a, S(\mu, P)) < n(c, S(\mu, P))$.

Definition 4.11 *Given approval index* μ, *the refined approval voting is a SCC with* $RF_\mu(P) = F_\mu^{\gamma(\mu)}(P) \cap F_\alpha(P)\ \forall P \in \Pi^N$.

Remark 4.4 *Given approval index* μ, $RF_\mu(P) \neq \emptyset\ \forall P \in \Pi^N$.

Proof. Take any μ, and any $P \in \Pi^N$. Since $F_\mu^{\gamma(\mu)}(P) \neq \emptyset$, $\exists x \in F_\mu^{\gamma(\mu)}(P)$ with $n(x, S(\mu, P)) \geq n(y, S(\mu, P))\ \forall y \in F_\mu^{\gamma(\mu)}(P)$. Futhermore, from the definition of the finest (μ,c)-approval voting rule we have
$n(x, S(\mu, P)) \geq n(y, S(\mu, P))\ \forall y \in A$, implies $x \in \alpha(S)$. So $x \in F_\alpha(P)$. ■

Proposition 4.5 *For given* μ, *the refined approval voting* $RF_\mu(P)$ *is Maskin monotonic refinement of approval voting.*

Proof. Take any μ. Since finest approval voting rule $F_\mu^{\gamma(\mu)}(P)$ and approval voting $F_\alpha(P)$ are Maskin monotonic, by Proposition 3.1 we get the refined approval voting $RF_\mu(P)$ is also Maskin monotonic. Also from the definition of the refined approval voting; we know that $RF_\mu(P)$ is a refinement of approval voting. ■

Claim 4.1 *For given μ, the refined approval voting $RF_\mu(P)$ is a minimal Maskin monotonic refinement of approval voting.*

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