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Forecasting and Modelling of Electricity Prices by Radial Basis Functions: Turkish Electricity Market Experiment

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Abstract.
In the past three decades the electricity has ceased to be a public good and was deregulated in many countries including the developing ones. Therefore the short-term price forecasts have become very important for all the market players in a new competitive environment. In this paper the hourly data of the Turkish electricity market are examined, considering the market players are interested in the evolution of hourly electricity prices and the aim is to forecast the hourly prices as close as possible to the market price to produce proper cash flow estimations. It is shown that the hourly prices have a recurring structure in time and this effect forms a rational for Radial Basis Functions method besides the conventional linear regression method in researching the price structure. Radial Basis Functions method produces slightly better estimation errors in terms of out of sample performance for a specific estimation period.
Keywords: *Radial Basis Functions, Electricity Price Forecasting, Energy Markets.*
JEL Classification: *C14, C53, Q41.*

Özet. Radyal Tabanlı Fonksiyon Ağları ile Elektrik Fiyatlarının Modellenmesi ve Tahmini: Türkiye Piyasası Uygulaması
Elektrik piyasası son 30 yılda gelişmekte olan ülkeler de dâhil olmak üzere pek çok ülkede özelleştirilmiş ve elektrik bir kamu yararı olmaktan çıkarılmıştır. Bu nedenle kısa dönem için elektrik fiyatı tahmini rekabetçi bir ortamda iş yapan piyasa oyuncuları için oldukça önemli hale gelmiştir. Bu çalışmada önce zaman içerisinde aynı elektrik fiyatlarının tekrar tekrar gözlendiği gösterilmiş ve bu nedenle radyal tabanlı modellerin bu yapıyı açıklayabileceği tartışılmıştır. Daha sonra radyal tabanlı fonksiyon ağı ve geleneksel doğrusal regresyon analizi yöntemleri kullanılarak saatlik gün öncesi piyasa fiyatı tahmin edilmeye çalışılmıştır. Radyal tabanlı fonksiyon ağının örneklem dışı tahmin performansının az da olsa daha iyi olduğu gösterilmiştir.
Anahtar Kelimeler: *Radyal Tabanlı Fonksiyon Ağları, Elektrik Fiyatı Tahmini, Enerji Piyasası.*
JEL Sınıflaması: *C14, C53, Q41.*

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1. Introduction

During the past three decades a growing number of countries restructured their various regulated markets, particularly the energy markets. (See Weron, 2006 for a brief history of liberalization process in Europe and US markets) The main motivation behind the liberalization efforts is to enhance the efficiency gains due to increased competition or to reduce the monopoly power, particularly in the long run. However, the debate on the efficiency gains, incentives on the capacity investments and also the future technical innovations are still questionable in the context of ongoing deregulation activities (See Kalaycı , 2002 for a critical analysis of Turkish electricity market liberalization). The electricity market has been one of the important deregulated sub-sector of energy business and attracted many researchers in the field. To a large extent, the aim of the researches has been to investigate and to explain the structure of the electricity market and to model the deterministic and the stochastic movements in electricity prices. By discovering the nature of price movements, the market players in spot and/or derivatives markets can obtain better predictions and can improve their bid and ask quotations which, in turn, may affect their firm's cash flows and the risk levels positively.

The large part of the literature on electricity modeling aims to forecast the electricity prices correctly in order to be able to produce quotations not only for spot markets but the derivatives markets as well. This issue has significant implications for cash flow management and hedging activities of the firms in this very newfangled business. One major difference between the models developed for spot markets and derivatives markets is the frequency of the data. The spot market forecasting is performed to predict the hourly prices in the day-ahead market, whereas the derivatives markets uses daily price indices and the forecasting models uses the average daily price data. Without doubt, some of the features of the hourly and daily data differ significantly as discussed below and therefore modelling for these purposes require different approaches. (Aggarwal, Saini & Kumar, 2009 is referred to the reader for a review and an evaluation on the forecasting of electricity prices in deregulated markets)

Electricity as part of an energy commodity markets has distinguishing characteristics in comparison with the other commodities. First of all it cannot be stored economically. (Hydro-electric units can be an exception to a reasonable extent) Then the electricity prices would be highly dependent on the electricity demand and the factors determining the demand for each and every time unit. In other words, business activity, temporal weather conditions including the snow and the precipitations and the similar seasonal parameters could affect the electricity prices significantly. (Lucia &Schwartz, 2002)

Secondly, it has a strong local character due to the capacity limits in the transmission lines and the transmission losses. This makes the interregional transmission of electricity uneconomical and the supply and demand characteristics are determined fully by the local conditions.

As a result of these distinguishing characteristics, the spot electricity prices may have an abrupt and infrequent changes due to the non-storability feature of electricity. These extreme changes are called “jumps” or “spikes” caused by transmission failures, weather conditions and possible generation outages. However, this “spiky” behavior is very short-lived and after a short period of time the prices revert to normal levels. This is called “mean reversion” as one of the stylized facts of electricity markets. The mean reversion feature can be observed in some financial markets, like interest rate markets in a weak manner, i.e. the speed of the reversion is not so fast, yet the electricity spot prices are expected to revert to normal levels quite rapidly in general. Besides this, when working with hourly data instead of daily ones, it is observed that the effect of mean reversion is quite vague and the persistency of the data outweighs the reversion effect. This issue is addressed in the later section of the paper.

The same argument also affects the volatility assessments. On one hand, electricity prices exhibit very high volatilities, say on a daily basis, in comparison with the other financial markets, on the other hand using the hourly data increase the volatility further than the volatility of daily data.

Pindyck (1999), examines the long run evolution of the energy commodities in an econometric context and Gibson & Schwartz (1990) and Hilliard & Reis (1998) examine the commodity contingent claims in a stochastic framework under two and three factor models respectively. Particularly for contingent claims on electricity, Clewlow & Strickland (1999) use mean reversion and Deng (2000) uses mean reversion, jumps and regime switching with the Fourier transform methods. Jong (2002) also produces specific option formulas with the model parameters determined by the mean reversion and jump processes. The spiky behavior and the regime switching models are also examined in Huisman & Mahieu(2003), Geman & Roncoroni (2006), Haldrup & Nielsen (2005), Bierbrauer, Menn, Rachev & Trück (2007), and Weron (2009), particularly in the context of determination of model parameters by using the regime switching in addition to mean reversion and the jump processes. The data of all the models quoted above are, without exception, daily data which serves to the specific aim of the referred papers. These specific aims are to produce the model parameters to estimate the spot and forward electricity prices for derivative markets. Conejo, Plazas, Espinola & Molina (2005) is one of the papers forecasting the electricity prices on hourly basis by using ARIMA and wavelet methods and Crespo, Hlouskova, Kossmeier & Obersteiner (2004) uses linear univariate time series for the same category of data. Huisman, Huurman & Mahieu (2007) focuses on hourly specific mean reversion for each hour by using panel model. Other than the parametric group of models the literature contains non-parametric group of models on forecasting electricity prices. Vahidinasab, Jadid & Kazemi (2007), Szkuta, Sanabria &Dillon (1999) and Voronin & Partanen (2014) are the few, particularly dealing with the artificial neural network applications in non-parametric field.

The prices and the electricity demand in Turkish market have been considered in Ozturk & Ceylan (2005) and Dilaver & Hunt (2011) in genetic algorithm and time series framework respectively. Ediger & Tatlidil (2002) focuses on the analyses of the cyclical patterns in Turkish electricity market. The literature is mainly concerned with the total and industrial electricity demand and deals with the forecasting of the future demand within a planning perspective.

However, in this paper the hourly data of the Turkish electricity market are examined, considering the market players are interested in the evolution of hourly electricity prices and the aim is to forecast the hourly prices as close as possible to the market price to produce proper cash flow estimations rather than determining the parameters of models based on the average daily prices. To pick the best approach among alternatives, we first analyze statistical properties of the hourly priced data in section 2. It is shown that the hourly prices have a recurring structure in time and this recurring and clustering effect forms a rational for such an analysis. For example, the data set has 709 different prices in 3 years ago for one year's time (with 8760 observations) and 3190 different prices in 3 years' time (with 32856 observations). This means that there may be some price ranges recurring themselves during time and through this property the price clusters may be constructed. Therefore it is considered that radial basis function networks (RBFN) can be used to explain this clustering effect and the relationship between the hourly electricity prices and the independent variables. Section 3 develops and analyses the basic deterministic linear model and the radial basis function model in such a framework. In Section 4 the empirical findings of the models and the out of sample performance of the models are discussed. The concluding remarks are given in Section 5.

2. Data and Statistical Properties

In this paper, the Turkish Electricity Spot Market price data (hereafter it will be called as PMUM or PMUM prices) are examined. PMUM is a "physical" market like "Elspot" and "APX" for Nordic Countries and UK respectively, where the spot electricity contracts are traded for physical delivery for each one of the 24 hours during the following day. These prices are called day-ahead prices or system prices which are determined by the supply and demand forces for each hour for the following day (Lucia & Schwartz, 2002). In other words these prices are the market clearing prices for all the participants making the market when there is no other imbalances or constraints in the system. In the presence of differences between the planned and the realized exchange of electricity load, i.e. in the occurrence of imbalances, system operator PMUM, like in the other markets, set up regulating and balance markets in a quite short span of time and the balance is restored through the price mechanism on a continuous basis. The prices used in this set up are called system marginal prices or system balancing prices. From time to time, it is observed that the market clearing prices fall down to zero and even at negative levels in some markets. (See the classification of the distinct characteristics of electricity prices in Knittel & Roberts, 2005) This may particularly produce difficulties when handling with the log-price data. However, there has been no negative price in the PMUM data referred to herein.

As pointed out in the introduction part, the structure of market clearing prices (day-ahead prices) of Turkish Electricity Spot market are explained on an hourly basis in this study. The total sample size contains 32856 hourly observations and covers the period between 01.01.2010-30.09.2013. (See figure 1) Yet, last 3 months' data (92 days) are set aside for out of sample performance and not included in the main data for estimation purposes.

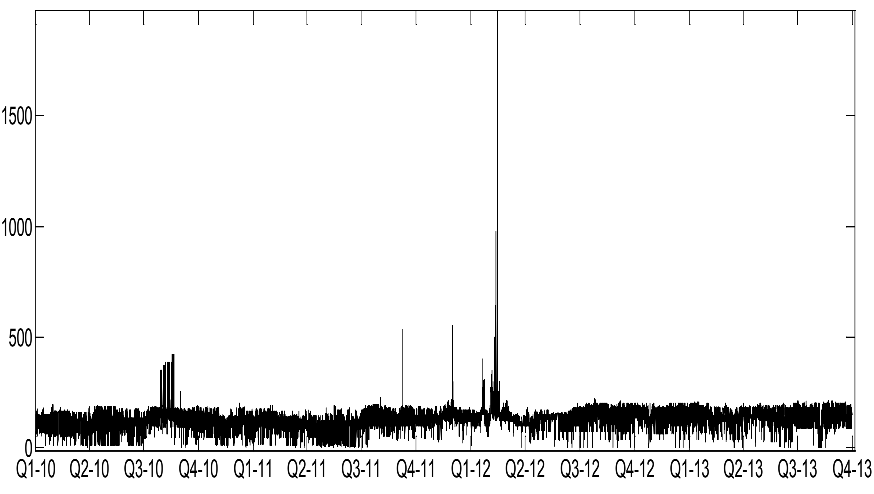

Figure 1: Hourly Spot Prices for the period 01.01.2010-30.09.2013 (TL/MWh)

As it can be observed from the figure, the price data have very limited amount of erratic behavior and exhibits quite stable properties. However it is also difficult to recognize the seasonality characteristics in the data. Figure 2, makes a zoom in the data and exhibits the daily frequencies more closely. It is easily observed that the daily price structure is almost persistently repeated every day on an hourly basis. The periodicity in the data can be seen very easily but the seasonal periodicities are hard to observe visually from the hourly data. This will be done by using the dummies later in the section. Yet, the monthly and particularly the weekly presentation of the data in Figure 2(a) and Figure 2(b) pronounces its most distinctive periodical characteristics. It can easily be seen from the charts that the hourly prices present a cyclical behavior and varies almost each hour during the day as a result of the structure of electricity markets and the demand of electricity. This is the fact almost in all electricity markets since the electricity load demanded is different within the various fragments of the day and it is one of the aims of this paper to capture this property.

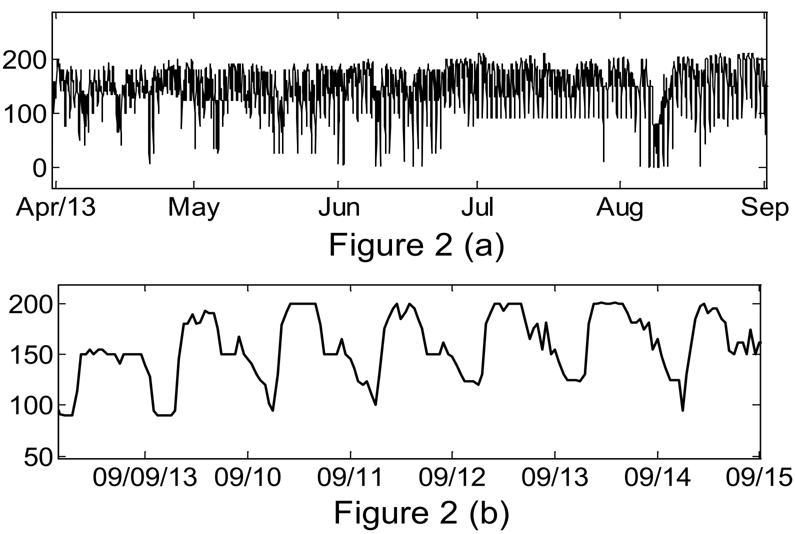

Figure 2: (a) Seasonal Characteristics of the Data- Monthly wise. The chart covers April 2013-September 2013 period only, (b) Seasonal Characteristics of the Data- Weekly wise. The chart covers 14 September 2013- 22 September 2013 week only.

The figures 3(a) and 3(b) contributes to discover the same relation with another perspective through a scatter plot. Figure 3(a) shows the electricity prices versus the electricity load for 3 main sub-division of the day time. These time spans can be classified as “normal” times, “peak” times and “slack” times depending on the electricity consumption and demand during the day. As can be seen from the chart easily, the prices get higher during the peak hours and fall down to lower levels as the demanded load turn back to the normal and lower levels. Similarly, the figure 3(b) captures the same relationship in monthly terms. However, the relationship between the electricity load and the prices are not so clear from a monthly perspective. It may be claimed that in spite of having the higher prices and loads during the summer months the same higher price levels can be encountered in different months irrespective of the season. In other words, the clustering in the price data can be regarded as vague for the months but may be significant for the specific hours of the day. As it can be seen from Figure 1(b) and Figure 3(a) the clustering of the day-time prices are more pronounced and this needs to be tested in the models referred in Section 3.

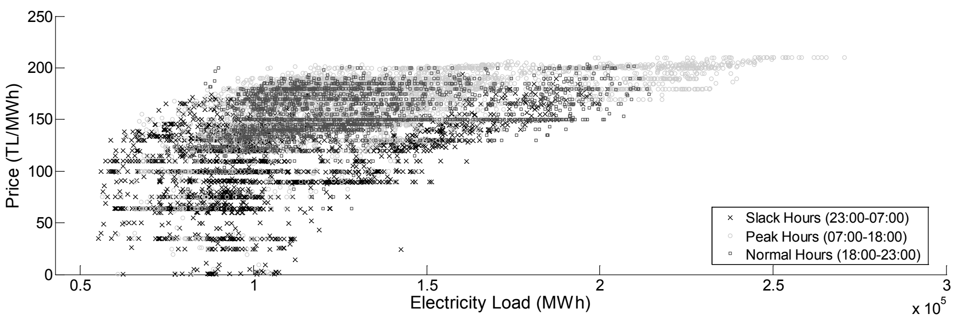

Figure 3(a): Price-Load Chart for 3 sub-groups of day; “slack”,” normal” and “peak” hours, covering the 01.01.2011-30.09.2013 period

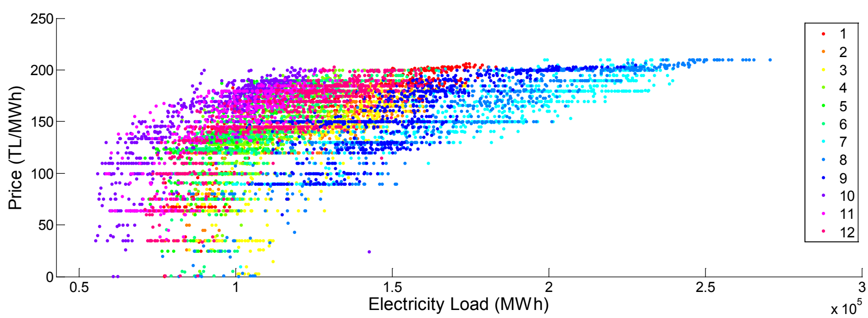

Figure 3(b): Price-Load Chart for 12 months covering the 01.01.2011-30.09.2013 period

The whole sample can be observed in Figure 4, covering the period 1.1.2010-30.09.2013 in a scatterplot.

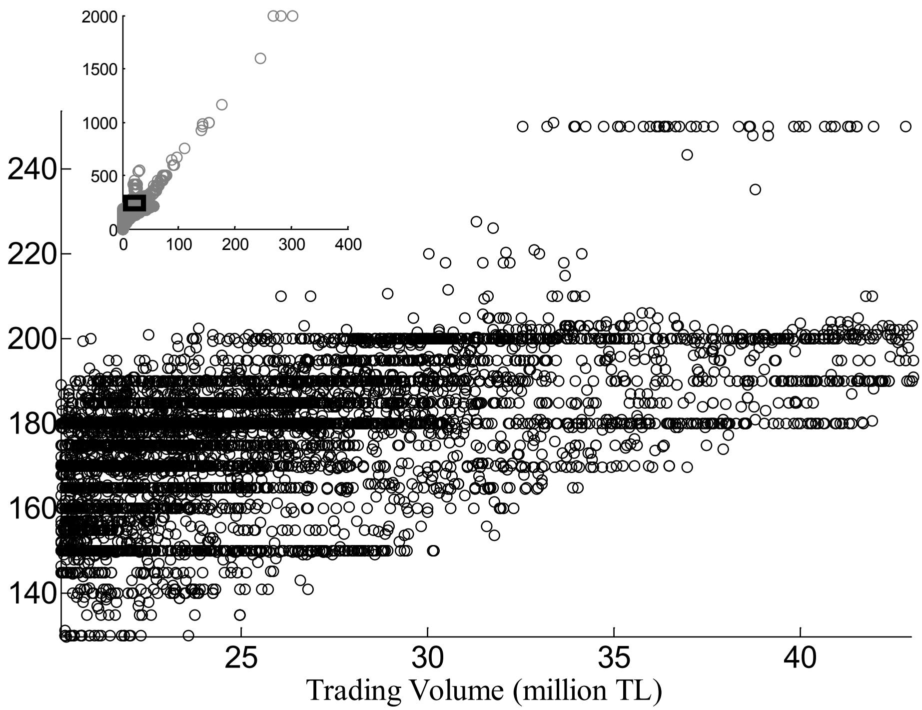

Figure 4. Clustering effect: Price-Trading Volume scatter plot of full sample space covering 01.01.2010-30.09.2013. The bigger chart zooms the range mentioned in the small rectangle in the chart at the left-top corner. Vertical axis shows the price levels.

In addition to the visual check of the data, the statistical properties of the hourly prices are given in Table 1 below. The hourly spot system price P_t exhibits a stationary characteristics as inferred from the ADF (Augmented Dickey-Fuller) statistics and the non-stationary test indicates that null hypothesis of unit root is rejected at % 1 level. This finding paves the way of working with price data itself, rather than the return data, without differencing the series or log-series.

Table 1: Descriptive Statistics for the Hourly System Prices and Other Related Prices Series.

The sample period is from January 1, 2010 to September 30, 2013. ADF denotes the Dickey-Fuller *t*-test statistics for the null hypothesis of a unit root and MacKinnon critical values are; -3.4304 at the 1% level, -2.8614 at the 5% and -2.5668 at the 10% level.

Series	Mean	Median	Min	Max	Std. Dev.	Skew	Kurtosis
P_t	135.076	140.00	0.00	2000.00	51.1	6.01	204.15
lnP_t	4.8	4.94	-6.92	7.60	0.65	-6.75	93.50
$lnP_t - lnP_{t-1}$	0.000	0.00	-11.51	11.78	0.45	0.47	274.27
Hour 1	138.77	140.00	11.68	199.99	27.55	-0.61	3.69
Hour 2	121.72	129.85	9.75	195.00	34.88	-0.70	3.25
Hour 3	107.04	115.99	0.00	191.42	40.23	-0.60	2.79
Hour 4	91.37	99.99	0.00	185.01	42.82	-0.31	2.23
Hour 5	86.20	91.01	0.00	180.00	42.78	-0.25	2.10
Hour 6	82.40	90.00	0.00	190.00	40.03	-0.34	2.18
Hour 7	78.90	82.60	0.00	159.94	40.69	-0.24	2.10
Hour 8	103.74	114.99	0.00	183.12	40.83	-0.76	2.83
Hour 9	138.14	145.00	3.00	203.00	37.78	-1.23	4.58
Hour 10	152.30	159.99	11.20	756.10	39.16	2.12	45.72
Hour 11	162.64	166.71	11.99	2000.00	59.96	21.39	648.13
Hour 12	170.11	170.50	55.03	2000.00	64.81	18.42	486.19
Hour 13	154.28	156.99	11.90	1162.77	43.61	9.29	213.91
Hour 14	158.24	162.00	11.90	1600.04	52.24	15.60	427.39
Hour 15	163.25	166.00	11.80	2000.00	64.54	17.34	482.59
Hour 16	157.13	161.90	11.50	999.01	45.64	5.37	93.64
Hour 17	154.61	160.00	11.70	999.00	42.97	5.43	112.52
Hour 18	149.81	153.00	10.00	925.87	48.18	3.90	58.89
Hour 19	143.46	148.97	11.35	952.13	45.78	5.08	86.91
Hour 20	141.42	143.00	10.00	599.17	36.79	1.67	25.70
Hour 21	143.54	145.90	40.00	450.05	30.02	0.34	10.88
Hour 22	139.37	141.01	52.04	250.01	29.88	-0.40	2.89
Hour 23	157.56	161.00	66.00	250.01	24.36	-0.49	3.24
Hour 24	145.82	149.95	10.00	200.01	27.43	-0.66	3.52

	ADF
P_t	-10.861
lnP_t	-12.245
$ln(P_t / P_{t-1})$	-41.889

The high volatility encountered in the price data is an essential characteristics of the electricity markets. As it can be noticed from the descriptive statistics, the hourly volatility of the percentage changes (log price difference) is 45.4 % which corresponds to an extremely high annualized level of 4249 %. This peculiarity stems from the fact that the hourly prices change constantly during the hours of the day depending on the level of business activity, day light and the similar factors and naturally increase the total variability in the system. For comparison, the volatility of log price changes in the Nord Pool on an hourly basis, yields % 6.77 which translates into an annualized level of 633.5 % which is almost one-seventh of the total variability in Turkish market.

Besides the continuous hourly changes in prices, the spikes may have a significant effect on the level of volatility as can be seen from Figure 1. There are some up and down movements in the figure, however the prices in the chart appear to have a tendency to revert to some level eventually and not much number of significant jumps are encountered.

Another interesting point deserving further explanation is the changing volatility structure of hourly prices in the data. As the descriptive statistics summarized in Table 1 shows, each individual hour may have a different volatility levels and this feature may produce better results than the traditional GARCH type models in terms of the predictions and the corresponding cash flow estimations. However, neither the different hourly volatilities nor the GARCH type models did not produce lower mean square errors for out of sample performance and the results are not reported in Section 3 and Section 4. For the impact of volatility on the predictability of market prices the reader is referred to Haugom, Westgaard & Solibakke (2011) and Garcia, Contreras, van Akkeren & Garcia (2005).

Last but not the least, the mean reversion feature of the electricity data is questionable when the frequency of the data is different from the daily ones. On the basis of hourly data, it is widely accepted that the price series exhibit some level of persistency to the changing level of lagged variables and do not display a strong mean reversion characteristics. This can be checked by various tests (Weron, 2006 p.50-52) and R/S analysis has been applied in this paper. Accordingly, the Hurst parameter (self-similarity parameter) has been founded as 0.70 for hourly data which shows the non-existence of mean reversion feature as expected. The values equal or lower than 0.5 would be indicating the presence of mean reversion effect.

Keeping in mind that the mean reversion effect and the related parameters are considered important in the various derivatives models the mean square errors have been reported in Section 4 for comparison purpose, yet the model itself is not explained in detail. (For the model and estimation method see the details in Lucia & Schwartz, 2002)

It must be pointed out that there is one important structural distinction between the mean reversion models on the basis of daily average price data and the hourly data. When it is used with the average data, no information is lost and the needed information is the average price of one-lagged electricity price structurally. However, when it is used with the hourly data, due to the structure of mean reversion equation all the remaining 23 hours of price data is lost and this slightly reduces the explanatory power of the equation compared to the linear model. As a result the model has lower R^2 and a higher regression variance in comparison with the basic linear model and this explains the exclusion of the model in this paper.

3. Models

3.1. Basic Linear model

In this section, the dynamics of the spot system prices is examined and discussed. For this purpose the basic linear model and equation is considered to be suitable and fits the hourly data better than the other stochastic models applicable for average daily price data. The assumed linear equation is

$$y_t = \alpha + \gamma t + \sum_{i=1}^{p} \theta_i y_{t-i} + \sum_{i=1}^{q} \delta_i X_{i,t} + \sum_{i=1}^{6} \epsilon_i DD_{i,t} + \sum_{i=1}^{11} \vartheta_i DM_{i,t} + \sum_{i=1}^{23} \beta_i DH_{i,t} + \varphi DB + u_t \qquad (3.1)$$

where the variables with the first letter D denotes the dummy variables; days, months, hours and holidays respectively. The electricity price y_t is the dependent variable and the independent variables are considered to be the lagged series of y_t, time trend t and X_i's in addition to the dummy variables. X_1 is the hourly temperature degrees and X_2 is the square of the temperature degrees covering the same period and finally X_3 is the cosine function of time as $\cos\left(\frac{2\pi t}{L}\right)$ where L is the periodicity of the hourly data (24 times 365), so $q = 3$, and u_t is the error term. The model parameters are estimated by applying the ordinary least squares for the equation.

The weather temperature and the natural gas prices are assumed to be two important factors affecting the electricity prices. Unfortunately the natural gas prices were constant during the same period and it was not employed in the equations. However, the hourly weather temperature and its square were used as explanatory variables in the equation. It is observed that the square of the temperature degrees have reasonable impact on the electricity prices and probably reflects the convexity existed in the prices and the seasonality is partially reflected by the cosine function in addition to the dummy variables representing the seasonality in the model. The both coefficients in the equation are found statistically meaningful at %5 confidence level. Table 2 reveals the summary statistics of the equation. This is the equation providing the least Akaike information criterion among the other equations and has been selected accordingly.

As the table shows, the hourly electricity prices prevailing in Turkish market are persistently dependent on the past prices and the lagged values even may go back into the past further. In order to have a parsimonious size of equation the lagged variables have been truncated at 35. More than this number of lagged variables has very immaterial impact on R^2 and may be safely ignored. The effect of hours is as expected and justifies the visual analysis made in Section 2. All the coefficients and signs of hourly dummies are statistically meaningful and negative for “slack” hours including the noon time (12.00) and holidays (annual holidays; δ_3) and positive for the “normal” and ”peak” hours. The coefficient of hours are positive until at hour 22.00 and turns out to be negative at 23.00 as expected. As for the month dummies there are only spring months (ϑ ; for February, March, April and May) in the equations and the signs are all negative, probably displaying the cheaper (water) sources are in effect in those months. It almost overlaps the situation shown in price-load chart in Figure 3(b). The appropriate day dummies are Sundays and Saturdays ($\dot{o}_1$ and $\dot{o}_6$) of which the coefficients and signs are meaningful at 95% confidence level. The prices fall down at weekends as expected almost in all markets. Finally, it may be inferred that there is an inverse relationship between the level of weather temperature and the electricity price, yet the impact is not much significant in absolute terms. The coefficients of square of temperature and the cosine function are also statistically meaningful and they explain the convexity and the periodicity in the prices to a reasonable extent respectively. By choosing the equation with a smallest Akaike and Schwarz information criterion as the best one, the out of sample performance of the last 92 days data has been tested and the results are presented in Section 4.

Table 2. Summary statistics of Basic Linear Model: $y_t = \alpha + \gamma t + \sum_{i=1}^{p} \theta_i y_{t-i} + \sum_{i=1}^{q} \delta_i X_{i,t} + \sum_{i=1}^{6} \dot{o}_i DD_{i,t} + \sum_{i=1}^{11} \vartheta_i DM_{i,t} + \sum_{i=1}^{23} \beta_i DH_{i,t} + \varphi DB + u_t$ Includes 30615 observations after adjustments covering the period 01.01.2010-30.06.2013. All coefficients are statistically meaningful at %95 confidence level.

Coefficients of the Model	Value	t-Stats
α	12.5491	13.0296
γ	0.0001	9.5820
θ_1	0.7719	135.6689
θ_2	-0.0648	-9.0583
θ_3	0.1523	21.2799
θ_4	0.0191	2.6959
θ_5	-0.1083	-15.1094
θ_6	0.0390	6.3061
θ_8	0.0186	3.1003
θ_9	-0.0255	-4.1921
θ_{11}	0.0205	4.8919
θ_{15}	-0.0235	-4.1898
θ_{16}	0.0146	2.1800
θ_{17}	0.0158	2.2730
θ_{20}	-0.0246	-5.6596
θ_{23}	0.0609	10.6623
θ_{24}	0.2138	30.6298
θ_{25}	-0.1373	-19.2076
θ_{26}	-0.0371	-5.1958
θ_{27}	-0.0638	-10.3557
θ_{29}	0.0123	1.9991
θ_{30}	0.0229	3.7939
θ_{33}	0.0184	4.4621
δ_1	-0.2184	-3.6099
δ_2	0.0066	3.1701
δ_3	-1.3815	-3.1501
$\dot{o}_1$	-4.6487	-12.7147
$\dot{o}_6$	-1.8832	-5.3630

Coefficients of the Model	Value	t-Stats
ϑ_2	-3.2983	-6.7221
ϑ_3	-3.6019	-6.4313
ϑ_4	-3.3111	-5.7463
ϑ_5	-3.8532	-6.8171
β_1	-10.7074	-14.3813
β_2	-11.2692	-14.8184
β_3	-12.7661	-16.4157
β_4	-7.6601	-9.7908
β_5	-5.7392	-7.2318
β_6	-7.8924	-9.9415
β_7	11.5743	14.3567
β_8	21.2956	26.5147
β_9	12.9978	15.8605
β_{10}	14.2563	17.4555
β_{11}	12.6882	15.3372
β_{12}	-3.5832	-4.3432
β_{13}	11.8953	14.3581
β_{14}	10.0767	12.4061
β_{15}	3.2371	4.1082
β_{16}	5.6554	7.6578
β_{20}	4.4612	6.3862
β_{22}	16.8533	23.6356
β_{23}	-2.0683	-3.1050

Adjusted R-Squared	0.8332
Std. Error of Regression	21.06

3.2. Radial Basis Function Network Model

This section uses the radial basis function networks to predict the hourly electricity prices in day ahead PMUM market. Radial basis function networks aim to approximate the target function using a combination of several local functions which are selected among a special class of function called, radial basis functions. A radial basis function (See, Yousef & Hindi, 2008 and Kon & Plaskota, 1998 for the description of the functions) is a function of a radial or a distance from some other point which is called a center. Technically, the radial basis function (RBF) can be defined for the domain D where $\left(x_n, y_n\right) \in D$ and $h\left(\vec{x}\right)$ is based on the radial $\vec{x}-\vec{x}_n$ where $\vec{x}$ denotes a vector. $\vec{x}_n = 0$ case denotes that the center is simply the origin.

Each center defines the radial basis neurons in the hidden layer of the RBF Network. The center and width parameters of the radial basis function contained in these neurons determine the strength and the range of the responses produced by them. The output of the RBF network, which aims to approximate the target function, is the linear combination of these local responses calculated in the output layer. Although some other metrics are possible, the radial is usually defined in terms of Euclidean norm or distance and the standard form is expressed as follows;

$$h\left(\vec{x}\right) = \sum_{n=1}^{N} w_n exp\left(-\gamma \ \vec{x}-\vec{x}_n{}^2\right) \tag{3.2}$$

where γ is a real-valued parameter and $w_1 \ldots\ldots w_N$ are the weight parameters to be found by the simultaneous equation system. Now, considering the vector of independent variables contains a huge number of data, $\vec{x}$ can be grouped into K centers and $\ell = 1 \ldots L$ clusters and final form can be stated as

$$h\left(\vec{x}\right) = y_n = \sum_{n=1}^{N}\sum_{k=1}^{K} w_k exp\left(-\gamma \ \vec{x}_{n,\ell} - \vec{x}_k{}^2\right) \tag{3.3}$$

for N data points and L clusters where $\ell = 1 \ldots L$ is the partitions of the whole data points (N).

In matrix form

$$\begin{matrix} y_1 \\ y_2 \\ . \\ . \\ y_N \end{matrix} = \begin{pmatrix} \exp(-\gamma \ \vec{x}_1 - \vec{\mu}_1{}^2 & \cdots & \exp(-\gamma \ \vec{x}_1 - \vec{\mu}_K{}^2 \\ \vdots & \ddots & \vdots \\ \exp(-\gamma \ \vec{x}_N - \vec{\mu}_1{}^2 & \cdots & \exp(-\gamma \ \vec{x}_N - \vec{\mu}_K{}^2 \end{pmatrix} \begin{vmatrix} w_1 \\ w_2 \\ . \\ . \\ w_K \end{vmatrix} \tag{3.4}$$

where

$$\varphi = \begin{pmatrix} \exp(-\gamma\ \vec{x}_1 - \vec{\mu}_1{}^2 & \cdots & \exp(-\gamma\ \vec{x}_1 - \vec{\mu}_K{}^2 \\ \vdots & \ddots & \vdots \\ \exp(-\gamma\ \vec{x}_N - \vec{\mu}_1{}^2 & \cdots & \exp(-\gamma\ \vec{x}_N - \vec{\mu}_K{}^2 \end{pmatrix}$$

and $w_1 \ldots w_K$ can be simultaneously solved as long as the matrix $\varphi^T \varphi$ is invertible, depending on the parameter γ to be learned. For solution, the γ is fixed and the $w_1 \ldots w_K$ solution is found as a first step. Then the solution $w_1 \ldots w_K$ is fixed and the error term is minimized with respect to γ, by changing it until the error term converges to a certain value.

Although there are various learning algorithms for RBF networks, typically learning in the RBF network has two steps (see Bors, 2001 and Fu, 1994 for details): learning in the hidden layer and then followed by learning in the output layer. First arbitrary values for γ and the number of centers are selected. Then using these arbitrary values, the steps of RBF network is performed to get an initial solution. The first step involves determining the centers ($\vec{\mu}_i$) using different methods including regression trees (Kubat, 1998), clustering algorithms (Moody & Darken, 1989) and genetic algorithms (Billings & Zheng, 1995). The second step involves learning the weights (w_j) between the hidden layer and the output layer using methods like least mean square or maximum likelihood estimator. Finally, γ is optimized for the solution obtained following these steps.

As partially mentioned in the introduction section, the reason why the radial basis function model is considered to be the suitable model for estimating the hourly electricity prices can be seen visually in Figure 5. In this figure the axes show the TL 0.1 price intervals between minimum and the maximum electricity prices (TL 0 and TL 2000 respectively) and the number of observations in these price ranges for the same period in 3.1. The first graph plots the whole price bins, while the second graph focuses the price bins between 0 and 250 TL, where most of the data are observed. Figure 5 supports our observations in Figures 3 and 4 in the previous sections that hourly electricity prices are repeated frequently and can be grouped using exogenous variables such as hours and months. In other words there may be found some price groups which might be represented by local functions using the distance from the average of explanatory variables in a range determined by a center. After detecting such a structure the prediction of hourly electricity prices may be performed by using the RBFs.

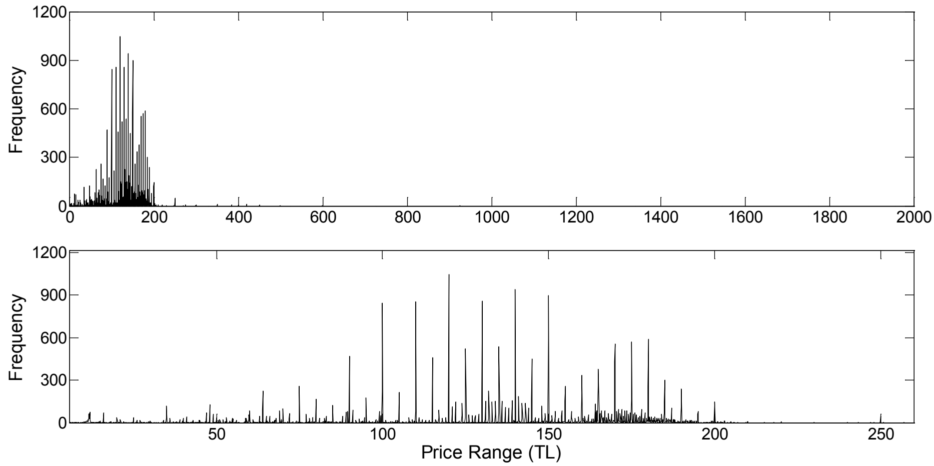

Figure 5: Price Range-Frequency Chart for the entire period.

In this application, the normalized hourly electricity prices have been chosen as the value of function $h(\vec{x}_n)$ i.e., y_n and the set of normalized independent variables in Equation 3.1 as x_n. Instead of a typical learning algorithm for RBF network, this study uses a different approach. One of the main concerns of RBF network is the number of clusters and setting the centers of these clusters. Since the price data in the Turkish Electricity Market are clustered as observed in Figures 3, 4 and 5, the clustering in this study has been based on the price data instead of independent variables according to the frequencies plotted in Figure 5. Instead of setting the number of clusters, we set the minimum number of observations in a TL 0.1 bin and define any bin with more observations as a cluster. If the number of observations in the bin exceeds the threshold, for example 40, this interval is accepted as a cluster. In this way as the number of frequency is kept lower, much higher number of clusters is generated and lower number of clusters is produced for the large number of frequencies.

The second difference of this study involves selection of γ. Instead of optimizing γ after obtaining an initial solution, our approach aims to learn all parameters in one step simultaneously. We run the RBF network for different values of minimum cluster threshold (between 30 and 200 with increments of 10) and gamma ($\gamma = 10^{-x}$ where x is between 0 and 10 with increments of 0.3) and pick the best fitting number of clusters and gamma using 10-fold cross validation. (Ten different estimations for each cluster)

Here is how the procedure works. First, the dataset is divided into 10 equal subsets randomly. Each time, one subset is hold out as the test set and the other remaining 9 subsets are merged to form the training set. Then we run the RBF network algorithm for each of these training sets (10 times) and calculate error for the subset which is used as the test set. Next, we calculate the average of the errors for these test sets to estimate the out-of-sample performance of the learning algorithm with selected γ and RBF center threshold values. In other words we rerun the RBF network learning algorithm 10 times for each training and test sets created for each γ (34 different gamma values between 0 and 10 with 0.3 increments) and RB center threshold (18 different threshold values between 30 and 200).

The steps of the algorithm is summarized below;

1. Formation of Cross Validation subsets: The dataset between 01.01.2010-30.06.2013 (30648 hourly observations) is divided into 10 subsets randomly and each subset is used as the test set once and as a part of the training set 9 times while the other subsets are used as the test set.

The following steps are repeated 10 times for each training set and test set formed in part 1;

1.1. The training set and test set is normalized using the standard deviations and the means of the training set.

1.2. After the determination of clusters, the averages of $\vec{x}_{n,\ell}$ are computed within each cluster as the center of each $\vec{x}_{n,\ell}$. Here the $\vec{x}_{n,\ell}$ contains 29 lagged prices, temperature and the square of temperature, 6 day dummies, 23 hour dummies,11 month dummies and time trend, totaling to 72 independent variables.

1.3. The number of observations in TL 0.1 price ranges are determined. Then the clusters are formed and the set of centers ($\vec{\mu}_i$) for the threshold given are calculated.

1.4. The weights (w_j) in Equation 3.4 is calculated for given value of γ

1.5. The predicted prices are calculated by post-processing (inverse normalization with parameter calculated in step 2.1) calculated in the values obtained from the equation 3.4 and the mean squared errors are calculated for the test set.

2. The average MSE is calculated as the expected value of out-of-sample error of the learning algorithm for given values of γ and the RB Center threshold.

The steps 2 and 3 are repeated for each γ and RB Center Threshold value set above. The average MSE calculated for these parameters are plotted in Figure 6. As it can be observed from the figure, the cross validation MSE are not much sensitive to the γ for values $x > 3$. The minimum cross validation MSE is obtained for $\gamma = 10^{-3.9}$ and 60 for the RB Center threshold (minimum number of frequency in a cluster). The number of clusters with 60 threshold value is 100. The R^2 value for these parameters using the dataset between 1.01.2010-30.06.2013 (30648 hourly observations) is 84%. The R^2 is not used to pick the best model since it gets higher and higher as the number of clusters increases. Practically it would be possible to reach up to higher R^2 values by increasing the number of clusters and so putting less number of prices into each bin, however in such a framework the model would have been ill-posed, because we would have also modeled the noise instead of the price signal itself and then increase the possibility of higher out of sample error.

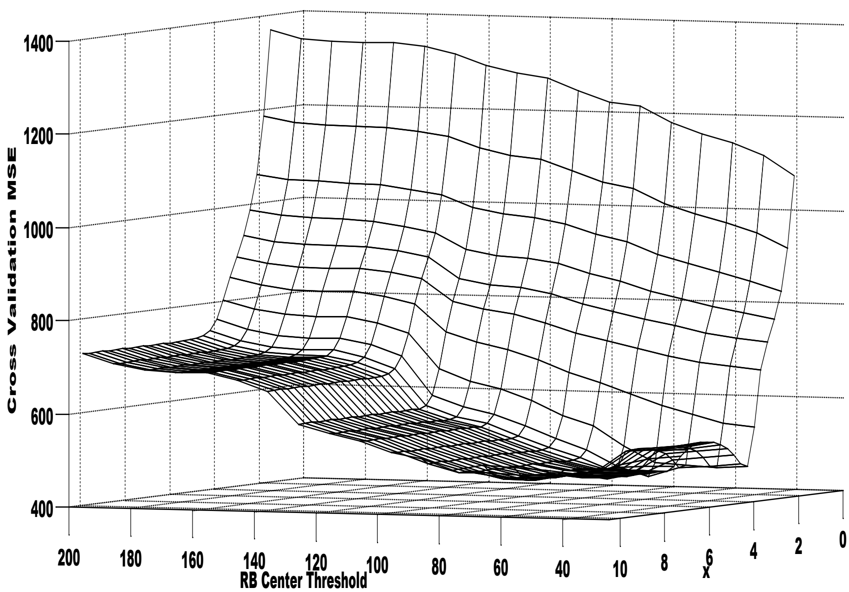

Figure 6: Distribution of Cross Validation MSE for the RB Center Threshold range between 30 and 200 and $\gamma = 10^{-x}$ for values of x between 0 and 10 with increments of 0.3

4. Out of Sample Performance

Finally, the findings of the RBF model have to be compared with the basic linear equation model's results. (See Weron & Misiorek, 2008 for the comparison of parametric and semi parametric time series model in forecasting framework)

For comparison the out of sample performance of both models, the period of 01.07.2013-30.09.2013 is used[1]. The results are reported in Table 3. The forecast made for this period by using the equation in 3.1 produces MSE of 147 TL and MAE of 8.44 TL.

Table 3: Out of Sample Performance of Basic Linear Model and Radial Basis Functions Model for the period of 01.07.2013-30.09.2013 (MSE: Mean Square Error, MAE: Mean Absolute Error)

	Basic Linear Model	RBF Model
MSE (TL) (Hourly Prices)	146.94	142.78
MAE (TL) (Hourly Prices)	8.44	8.39
MSE (TL) (Daily Averages)	14.68	12.04
MAE(TL) (Daily Averages)	2.61	2.13

As it is observed from Table 3, the RBF method provides better results compared to the linear model in terms of MSE and MAE. As a consequence of this, it is important to bear in mind that, particularly for the hourly data, it is not always the conventional methods that yielding the better performance and some other non-parametric methods like RBFs, may capture the market structure better than the conventional parametric methods.

1 The forecasting work has been carried out with the known temperature degrees for the out of sample period and no different econometric equation has been estimated for the weather temperatures. Undoubtedly, such an equation would be necessary for the forecasting of future periods longer than mentioned in this paper.

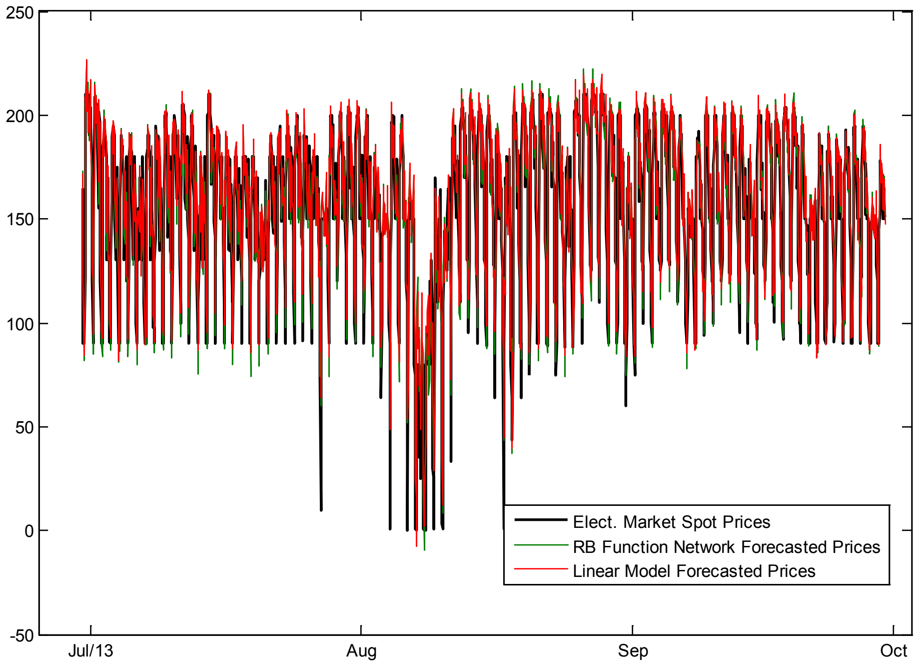

Figure 7: Forecasted prices vs. Observed Prices for July-September 2013 period on an Hourly Basis.

On top of that the out of sample performance of both models has been calculated based on the daily average prices which are used particularly by the derivatives markets as expected spot prices. In other words the forecasting of daily average prices can be approached by forecasting of hourly prices. In this case, naturally, the MSE and MAE values fall significantly due to the averaging property and the estimation errors in terms of absolute error in Table 3 are 2.61 TL/MwH and 2.13 TL/MwH for linear model and RBF model respectively.

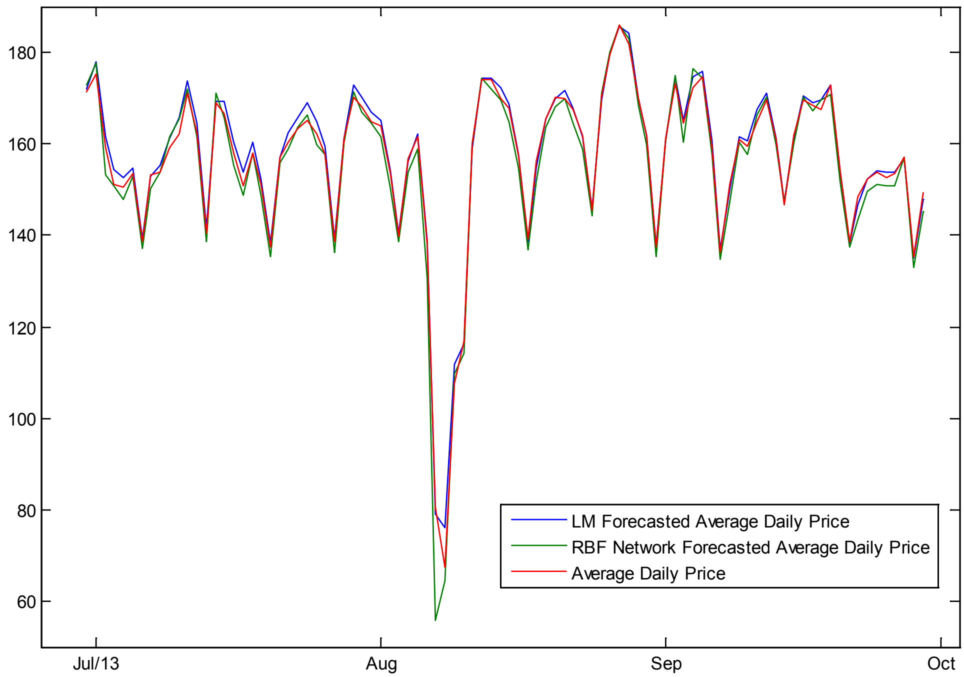

Figure 8: Forecasted daily average prices vs. Observed Daily Averaged Prices for July-September 2013 period.

The forecasting performance of both models is visually given by Figure 7 and Figure 8. It can be observed that the forecasts of both models have been almost overlapping and following the same recurring structure. Although the RBF method has been producing better results for spot and derivative markets, it may be said to have a disadvantage over the basic linear model in terms of the forecasting time it required. The cross validation work, on one hand requires to split the data into groups and re-run the estimation algorithm many times for each group of data, on the other hand it requires to run the same algorithm for each gamma value and the number of clusters as mentioned in Section 3.2.

5. Conclusion

This paper aims to explain the price structure in Turkish electricity market based on the linear regression method and RBF method using the hourly data. The empirical findings show that both methods explain the price structure well and consistent predictions can be made on an hourly basis. Although the RBF method appears to be slightly better in terms of out of sample performance the time and maintenance cost of the method may outweigh the simplicity advantage of the basic linear model. It can be mentioned that there is a tradeoff relationship between these two models; on one side the RBF produces relatively less estimation error, on the other side it causes more prediction time and effort with respect to the basic model. Whether it worth to replace the basic model by non-parametric models like RBF is the issue which may require further verifications by the traders, practitioners and market makers.

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