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CAN ALTUĞ SEÇİLMİŞ
117679017

ASST. PROF. ÖMER BEHİÇ ALBAYRAK

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Syntheticity of Mathematical Propositions in Kant's Theory of Knowledge

Kant'ın Bilgi Teorisinde Matematiksel Önermelerin Sentetikliği

Can Altuğ Seçilmiş
117679017

Advisor Asst. Prof. Ömer Behiç Albayrak
Istanbul 29 Mayıs University

Committee Assoc. Prof. Ferda Keskin
Istanbul Bilgi University

Committee Asst. Prof. Zeynep Talay Turner
Istanbul Bilgi University

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« *To make plans is most often a presumptuous, boastful mental preoccupation, through which one presents the appearance of creative genius, in that one requires what one cannot himself provide, censures what one cannot do better …* » 4:262–3

PREFACE

The present study is an attempt to examine Kant's classical distinction between analytic and synthetic judgments [in general] and the syntheticity of mathematical propositions [in particular] as developed definitively in the *Critique of Pure Reason*.

The topic has been quite commonplace in Kantian scholarship. It is safe to say that neither its importance nor its centrality has been overlooked. Nevertheless, while sometimes the merit of a study lies in the uncharted, sometimes it does in the exact opposite. For, especially in the domain of philosophy, such commonality indicates inexhaustibility rather than weariness. The distinction, in this respect, is a *classical* (4:270) philosophical problem *par excellence*. In this regard, achieving a *rational* grasp of the subject matter in place of that merely *historical* would be more than rewarding for the author of this paper, as is the case with all classical problems of philosophy, and it is in this very idea he takes refuge in attempting to the task here at the expense of rigor in a way unworthy for a study of Kant. To this end, attaining a first-hand insight into the subject matter in line with the tenets of critical philosophy, and thus merely providing a coherent and integral whole remains the chief concern.

In this regard, this study will be an exegesis, treating exclusively the original texts.[1] As such, it is concerned neither with covering a history of philosophy nor laying down a review of secondary interpretations.[2]

Just like one's thoughts follow in and through time only one after another, and never at once together, any text, too, has to follow an unilinear flow. This limitation requires that it is just as important to order the exposition on paper, for what precedes conditions what succeeds; and therefore, such ordering is not about mere style but rather about a deliberate arrangement as to deliver its content intelligible.

[1a] In addition to the *Critique of Pure Reason* (1781, 1787), mainly *Prolegomena* (1783) and *On a Discovery* (1790), taken in conjunction with relevant *Reflexionen*, correspondence, and lecture notes. The third *Critique* (1791) and *Opus Postumum* which would have reasonably considered for this study are alas not included in the scope.

[1b] Historical reach is restricted only to immediate *school metaphysics*, the Wolffian dogmatism (Wolff, Baumgarten, and Meier, on one end; Eberhard and Maaß, on the other) and relatively to what might be regarded as Leibnizian in Leibnizian–Wolffian school.

[2] Secondary literature is avoided, to the extent this is possible, for the purposes of exegesis, and no particular line of interpretation is readily subscribed to. Having said that, I do owe H. J. Paton the organization and the ease in *introducing* the real problem. I have also benefitted immensely from the editorial work done for the *Critique of Pure Reason*, namely, by Norman Kemp Smith, Werner Pluhar, and Paul Guyer and Allen Wood, as well as that for Baumgarten's *Metaphysica* and Meier's *Auszug* by their respective editors. I have also been helped by the presentations of Gottfried Martin and Henry Allison to *Philosophisches Magazin* and Schultz's *Erläuterungen*, respectively; and by the general instruction of Lewis White Beck in *early German philosophy*.

TABLE OF CONTENTS

ABBREVIATIONS

References to the *Critique of Pure Reason* (hereinafter be referred to as the "first *Critique*" or as the "*Critique*" alone) are cited as per the pagination of the original first (A) and second (B) editions; to *Reflexionen*, as per the number of respective item; and, to all other works of Kant as per the respective volume and page number assigned thereto in the Academy edition,[3] and are as follows:

1:387–416	New elucidation of the first principles of metaphysical cognition (1755)
2:65–163	The only possible argument in support of a demonstration of the existence of God (1763)
2:167–204	Attempt to introduce the concept of negative magnitudes into philosophy (1763)
2:275–321	Inquiry concerning the distinctness of the principles of natural theology and morality (1764)
2:387–419	On the form and principles of the sensible and the intelligible world (1770)
4:255–383	Prolegomena to any future metaphysics that will be able to present itself as a science (1783)
7:119–333	*Anthropology from a pragmatic point of view* (1798)
8:187–251	On a discovery whereby any new Critique of Pure Reason is to be made superfluous by an older one (1790)
9:1–150	*Jäsche-Logik* (1800)
10:129–35	Letter to Herz (February 21, 1772)
10:136–43	Letter to Garve (August 7, 1783)
10:344–7	Letter to Mendelssohn (August 16, 1783)
11:394–6	Letter to Beck (December 4, 1792)
20:257–332	What real progress has metaphysics made in Germany since the time of Leibniz and Wolff? (1793)
24:7–103	*Blomberg-Logik* (*ca.* 1770s)
24:87–118	*Hechsel-Logik* (*ca.* 1780s)
24:676–784	*Dohna–Wundlacken-Logik* (*ca.* 1790s)
24:787–940	*Vienna-Logik* (*ca.* 1780s)
28:531–94	*Metaphysik L2* (*ca.* 1790–1)
29:747–940	*Metaphysik Mrongovius* (1782–3)
29:943–1010	*Metaphysik Vigilantius* (1794–5)

[3] *Kants gesammelte Schriften*, ed. Königlich Preußischen Akademie der Wissenschaften (Berlin: Walter de Gruyter, 1902–1983).

ABSTRACT

Possibility of a priori cognition is the general problem of pure reason. Kant articulates and resolves this problem in terms of analytic–synthetic distinction of judgments. The distinction is underwritten by core metaphysical and logical doctrines in Kant's wider theory of knowledge. Syntheticity of mathematical propositions is examined against this background.

ÖZET

A priori bilme imkânı saf aklın genel meselesidir. Kant, meseleyi analitik ve sentetik hükümler bakımından ifade ve tahlil eder. Ayrım hâlihazırda Kant'ın bilgi teorisi çerçevesinde temel metafizik ve mantık öğretilerine dayanır. Matematiksel önermelerin sentetikliği bu bağlamda mülâhaza edilecektir.

INTRODUCTION

The *Critique of Pure Reason* emphatically pronounces the question "how are synthetic judgments a priori possible" (B19) as its real (*eigentlich*) problem (*Aufgabe*). The emphasis should not be taken for a literary device to underscore the importance of this question among other concerns, as the case may be; rather should it be read literally, in that, synthetic a priori is Kant's key philosophical device in *both* articulating and resolving the problem of the *Critique*.

The idea of synthetic a priori

To begin with, the question is formulated, and "with scholastic precision," (4:276) not as *whether* synthetic judgments are a priori possible but as *how* they are so—figuring that they already *are*. This formulation thereby indicates the Kantian hypothesis which is to constitute the basis of entire critical enterprise.

Now, the problem originally unfolds (A1/B1) with the premise that we are capable of a priori cognition.[4] Namely, not only we are able to establish certain judgments a priori, but we do also possess certain concepts of a priori origin. (B5) But then "what is especially remarkable" (A2) about this is that not only are we capable of a priori cognition, but our experience also would not be possible without it. Indeed, if we remove everything empirical from our experience, "there still remain certain original concepts and judgments which must have arisen entirely a priori" (A2) and contributed by "our own cognitive faculty out of itself." (B1)

[4] *Erkenntnis*. Although in previous [English] editions of the *Critique* (*i.e.,* Kemp Smith and earlier) this term along with *Wissen* and [occasionally] *Kenntnis* were all translated as 'knowledge,' it is now a standard and accepted practice to refer to these respectively, as cognition, knowledge, and acquisition [or information, according to the context]. In the present section of the study (as well as in its title) the term 'knowledge' is at places kept, in order to accentuate a possible tension between *Erkenntnis* and *Wissen* (§§3.2 and 3.2.1), and partly in concession to broader philosophical terminology, and partly to the fluency in narrative.

The fact that we actually do possess a priori cognition (*quid facti*) requires, before all else, its explanation as possible (*quid juris*). Accordingly, we could not "establish anything about objects before they [were] given to us," (Bxvi) in the first place, if our cognition were to conform to these objects. That is, under this assumption, we could not have accounted for a priori cognition. However, if we were to *reverse* this [traditional] assumption and to *hypothesize* that [it must be] the objects [that] were to conform to our cognition instead, this would have fared surely well with the [required] possibility of a priori cognition [of objects].[5] (Bxvi f.) Therefore, it is only under the latter assumption the possibility of our a priori cognition is warranted—hence the *Kantian hypothesis*, or the 'Copernican revolution' in philosophy.

Accordingly, it is by virtue of these a priori concepts and judgments which we contribute to experience, we can say "more about the objects that appear to the senses than mere experience would teach" and indeed make assertions "with universality and necessity which empirical cognition can never afford." (A2) For, indeed, "all we can know a priori [of things] is what we ourselves have put into [these things]." (Bxviii) Now, this is precisely the idea of a priori synthesis. "But what says still more" (A2/B6) is that some of our a priori judgments not only make the experience possible but are independent of *any* experience and expand our knowledge; and these are none other than synthetic a priori judgments.

Now, all the critical theses emerge in the extrapolation of this idea under Kantian hypothesis. For, if objects were to conform to our knowledge, then this entails *objects as we know* and, by extension, *objects as we cannot know*. That is, there must be a world of appearances which conforms to our knowledge and which we thereby determine. The world we know, the phenomenal world, is thus a world transformed by the transcendental mechanism of our mind. The world as it is, the

[5] Kant, at the beginning, (A1/B1 ff.) appears to address [mere] a priori cognition, however what he actually refers to is synthetic a priori cognitions, for a priori cognitions of or about objects can only be synthetic. Indeed analytic a priori cognitions pertain only to [mere] concepts.

noumenal world, on the other hand, is beyond our determination and thus unknown to us. It is also this noumenal world which warrants the possibility of [our belief] in God, our [postulating a] soul, our freedom, and ourselves as practical beings—*i.e.,* of those which we could not ever [possibly] *know*. Accordingly, the problems falling under the "central and revolutionary doctrine of the *Critique*"[6] are all interrelated, interdependent, and thereby can be considered as bound under *one* problem of synthetic a priori cognition, in that, they come about only under this idea. The *Critique of Pure Reason* thus takes up the question "how are synthetic judgments a priori possible" as its general problem under which its investigations can be brought (B19) and on which everything hinges. (4:276)

Synthetic a priori judgments are contained as principles in all substantial theoretical sciences of reason: pure mathematics, pure physics, and metaphysics. (B14) "The entire final aim of our [speculative] a priori cognition rests on such synthetic, ampliative principles." (A10/B13) Now mathematics and physics, which are in the secure path of science, provide a solid example thereof. (Bx–xiv) Metaphysics, albeit not yet in the secure path of science, *must* nonetheless contain synthetic a priori judgments in order to warrant its claims. (Bxiv f.) In this regard, the general problem of pure reason appears respectively in three questions: how pure mathematics is possible, how pure physics is possible, and whether metaphysics as a science is possible.

However, the case of mathematics is exceptional in respect of syntheticity. For propositions of mathematics are certainly a priori, for there is an evident epistemic difference between propositions of mathematics and, *e.g.,* judgments of experience. Nevertheless, it may be argued that mathematical judgments are analytic. (*e.g.,* B20 and 4:273) Conversely, with respect to natural sciences, the case is the other way around. That is, it is certain that principles of natural science are not an analytic, for they definitely say more about the objects than mere experience would. Still, their

[6] For a lucid discussion of the point, *see* Paton, 76 ff.

a priori status can be challenged, and it can be argued that such principles are actually derived from experience. (*e.g.,* B21 f.) This affinity is confirmed by the history of philosophy as well, in that it is predominantly agreed that mathematics is a priori and that natural sciences are indeed saying something more, while it is also not uncommon that the former is held to be analytic and the latter a posteriori. To summarize, among synthetic cognitions a priori, syntheticity of mathematics and apriority of natural science [and [secure] claim of metaphysics to truth] are not immediately evident. In this light, mathematics requires a closer inspection in respect of syntheticity.

The formulation of synthetic a priori

If no philosophical element stands in vacuum but inevitably in a [larger] system [of thought], this is certainly all the more so in the case of Kant and the architectonics of the *Critique*. [7] Accordingly, synthetic a priori is not *fiat*. That is to mean, the synthetic a priori is not posited arbitrarily in itself as a philosophical *deus ex machina* in order to simply overcome [this or that] problem. For, had this been the case, it would have merely been the designation of a problem and not a solution.

Now, it is a philosophical invention, to be sure, but an invention nonetheless 'according to principles,' to borrow the signature Kantian phrase. In this view, the idea of synthetic a priori finds its philosophical formulation in the doctrine of syntheticity. That is, it is through the distinction between analytic and synthetic judgments the problem is formulated and attempted a philosophical solution. In this light, the distinction itself merits a question of its own.

[7] Kant is a systematic (A855) philosopher, and the *Critique* is not a mere sum but an attentively thought entirety whose parts are interdependently related, thus cannot be isolated or separately amended (Bxxxviii). [Conjecturally] these parts are but different aspects of one and the same whole. There must be nothing, not belonging to the system (A474), hanging in vacuum, not grounded or connected. Blueprint of this aggregate is embodied in what is called architectonics. This systematicity is also the fulcrum which will allow the paradigm case to traverse the *Critique*. (A707)

> "But I wish I could persuade you ... to encourage an examination of my theses, considering them in the following order: One would first inquire whether the distinction between analytic and synthetic judgments is correct; whether the difficulties concerning the possibility of synthetic judgments, when these are supposed to be made a priori, are as I describe them..." (*Letter to Mendelssohn*, 10:345)

> "And so, my dearest sir, I beg you, if you should wish to apply yourself any further in this matter, ... [do not] grab everything or anything at all at once, out of context, but to consider the work in its proper order: first, to examine ... distinction between analytic and synthetic knowledge; then, to proceed to the consideration of the general problem, how synthetic a priori knowledge is possible ..." (*Letter to Garve*, 10:340 f.)

Responding to the plea of Kant, this study will examine how the formulation of syntheticity adequately grounds the idea of synthetic a priori in respect of the critical problem, and how it is grounded by the core logical and metaphysical doctrines in Kant's theoretical philosophy.

1. GENESIS OF THE DISTINCTION

While the analytic-synthetic distinction of judgments appears publicly for the first time in the *Critique of Pure Reason*, it was long in the making in parallel to the development of adjacent themes in Kant's thought.[8]

For that matter, the terms 'analytic' and 'synthetic' had already been coined and in use in early modern tradition, albeit in reference to scientific methodology; and were as such recognized by and had currency in the Wolffian school. According to this convention, analytic denotes the regressive method and synthetic denotes the progressive method of science, referring to the direction of procedure in a rational

[8] In this context, Kant's philosophical thought until 1781 can, for the purposes of convenience, be schematized crudely in five turns. *(i)* The first turn, *ca*. 1755–61, registers a *reconciliatory* period, in which, while being critical of the orthodox rationalist tradition, Kant still sustained both possibility and dogmatic method of metaphysics and sought to remedy its foundations by reconciling Wolff and Crusius (*New Elucidation,* 1755). *(ii)* The second turn, *ca*. 1762–4, can, on the other hand, be characterised as the *revisionary* period, for, while maintaining the possibility of an orthodox rationalist metaphysics on one hand, Kant endeavoured to revise its foundations with a particular concern to the method of metaphysics. Specifically, Kant articulated the Crusian distinction between the logical and the real, identifying the relation between cause and effect as a real relation; and proposed to import into metaphysics certain mathematical concepts which are otherwise unavailable to it (*Negative Magnitudes,* 1763). Kant finally thoroughly problematised the method of metaphysics by working out fundamental differences between metaphysics and mathematics (especially with respect to definition, *viz*., concept formation) and distinguished between the analytic method of philosophy and the synthetic method of mathematics (*Inquiry,* 1764). *(iii)* Third milestone, *ca.* 1766, brings about a *sceptical* period, where Kant was now expressly dealing with the possibility and cognitive claims of metaphysics under predominantly empiricist epistemological considerations. The ensuing criticism of metaphysical claims gave way to the acknowledgment of the limits of knowledge, that is, the limits which 'human understanding' is bound within and cannot transcend (*Dreams of a Spirit-seer,* 1766). *(iv)* The fourth milestone, *ca*. 1769–71, avowedly bringing a "great light" (R5037) and planting the seed of the future *Critique,* (10:345) marks the *semi-critical* period which represented "the culmination of Kant's early philosophical development" and in which Kant could develop some of the main critical theses, such as the theory of time and space as a priori forms of sensibility. At any rate, the crux was, true to its name, the formulation of the duality of the sensible world of phenomena and the intelligible world of noumena. In contrast with Leibnizian metaphysics, the distinction between the sensible and the intelligible was of kind and not of degree. This sharp division was grounded in two corresponding faculties of the mind, sensibility and understanding. However, still in line with Leibnizian metaphysics and at variance with the [later] critical doctrine, the concepts of the understanding had a real use in the domain of the intelligible (*Inaugural Dissertation,* 1770). *(v)* Finally, the fifth and the last turning point, the famous *silent decade*, *ca*. 1771–81, is inaugurated by "a recollection of Hume" and his problematization of causality which decidedly awakened Kant from his "dogmatic slumber" (4:260) and provided "the last spark of light" required for the *Critique*. This decade was thus the working out of the objective use and validity of the categories and marked by the contemplation of "a new book entitled *The Limits of Sensibility and Reason.*" (10:129)

inquiry in that analysis proceeds from givens to the direction of their principles, whereas synthesis proceeds from principles onto their grounds.[9]

In continuity with the tradition through the Wolffian milieu, Kant adopts these terms along similar lines, reworking and perfecting them, to distinguish between methods of mathematics and philosophy, and further, between the definitions employed in these sciences, respectively as analytic and synthetic. Accordingly, philosophy arrives at its concepts analytically, whereas mathematics forms its concepts synthetically. In philosophy the concepts [of things] are given and to be merely analysed as to attain distinctness, while in mathematics, the concepts [of magnitudes] which are already clear is [combined and thus] made. (*q.v.* §3.2)

As for the distinction of judgments, Kant's first notes with respect thereto date back to as early as 1764, emerging in marginalia to Baumgarten's *Metaphysica* and Meier's *Auszug*. His initial distinction is informed principally by that between the logical and the real (*Negative Magnitudes*, 1763), and the philosophical and the mathematical (*The Inquiry*, 1764).

During this period (1764–69), Kant holds a sharp division between synthetic and analytic judgments, maintaining that all synthetic judgments are empirical [or real, as to contain relations of things] and all analytic judgments are rational [or logical, as to contain relations of concepts], (R3738, R3744, R3747, R3814) which would translate, respectively, to a posteriori and a priori judgments. (*q.v.* §3.3.1) However, this strict schism was also disturbed by Kant himself in the very same notes in which he recognized simultaneously the concept of cause as synthetic *hence* empirical, (R3749) and mathematical propositions as synthetic (R3750) *but* a priori.

[9] *E.g.,* in this sense, *Critique of Pure Reason* employs synthetic method whereas *Prolegomena* employs analytic method. *See* 4:263.

In the year 1769, which Kant deems as marking a turning point in his philosophical thought (R5037) and his "first awakening" (4:338), the notes regarding the distinction become more extensive and prosaic, (R3914, R3923, R3928) still not fundamentally different from his initial characterizations. It is only as of 1772, that is, after his [final] 'awakening' by Hume (4:260), the first truly extensive treatment of the distinction appears along with its mature formulation, (R4477, R4634) which Kant now incorporates in his logic lectures in 1770s. (*e.g.*, 24:232 f.)

In light of these, it can be seen that, although Kant conceived the analytic-synthetic distinction of judgments in 1760s, it was only in the 1770s he could formulate the synthetic a priori. Accordingly, there are two points that call for particular attention and that should be noted as regards the evolution of the distinction.

First, while Kant had initially made the strict distinction between analytic and synthetic judgments respectively as rational and empirical [in more or less conformity to rationalist and empiricist orthodoxies which he was to shortly overcome (*q.v.* fn. 11)], he still conceded the syntheticity of [a priori] mathematical propositions. This exhibits an apparent tension [*inter alia*] which [must have] challenged the earlier schism and compelled Kant to account for the possibility of mathematical judgments as both synthetic and a priori. The treatment of this problem in its full generality is thus very likely to have contributed to the genesis of critical philosophy.

What is perhaps more significant about these notes is that although Kant distinguished between analytic and synthetic judgments as early as 1764, the distinction appears in none of his public works [*i.e., Dreams* of 1766, *Directions in Space* of 1768, and even the 'semi-critical' *Inaugural Dissertation* of 1770 in which Kant had already developed certain elements of his mature philosophy] until 1781. This effectively points out to the indispensability of the synthetic formulation of a priori in the critical doctrine. (*q.v.* §3.3.1) In short, it was as if Kant had already devised the key but not found the lock still.

2. SURVEY OF THE DISTINCTION

At the outset of the *Critique of Pure Reason,* Kant lays out two key distinctions that are to jointly and severally cut across and qualify human cognition. While the first set of these, a priori and a posteriori, which is primordial within the philosophical tradition, concerns the source of cognitions; the second one, analytic and synthetic, which is novel, (*q.v.* fns. 11, 44) pertains to their epistemic value.

2.1. A priori and a posteriori

Characterised negatively, a priori cognitions are those which are independent of experience. Counterposed to these are empirical cognitions which have their source [a posteriori] in experience. (B2 f.)

Still, there are also those cognitions which we can nevertheless *establish* a priori, however "from empirical principles" derived from experience. (29:750) In Kantian terms, these are not considered properly among a priori cognitions which must occur not just "independently of this or that experience" but "absolutely independently of all experience." (B2 f.) At that, a priori cognitions too can be either pure or non-pure as well: pure, if "nothing empirical is intermixed" therewith; non-pure, if there is in them something "drawn from experience."

Then, characterised positively, a priori cognitions are thought with strict necessity and true universality, that is, with "no exceptions whatsoever to the rule;" whereas empirical cognitions can [at most] give only assumed necessity and comparative universality, that is, they are thought with "no exception *yet* to this or that rule." While the former commands absolute certainty, the latter affords merely an arbitrary increase in validity. The positive criterion, according to Kant, serves as a "secure indicator" for ultimately distinguishing between a priori and a posteriori cognitions. (B3 f.)

Consequently, there are *(i)* a priori judgments (*simpliciter*), *i.e.,* cognitions that are established *absolutely* independent of all experience, whose composition may be either *(ia)* pure or *(ib)* non-pure; and *(ii)* a posteriori judgments, *i.e.,* cognitions that are established *ultimately* through experience, either *(iia)* as being contingent or *(iib)* with assumed necessity we are nevertheless capable of establishing a priori (*secundum quid*).[10]

[10] Prior to any further inquiry, however, a priori-a posteriori distinction needs a comprehensive delineation as to ascertain its indications precisely. This is all the more warranted considering the otherwise familiarity of the distinction would silently be more of a pitfall than convenience.

"Yet here we can content ourselves with having displayed the pure use of our cognitive faculty as a fact together with its indication. Not merely in judgments, however, but even in concepts is an origin of some of them revealed a priori." (B5) With this passage borne in mind, the distinction alludes throughout the *Critique* to *(i)* origin, that is, *(ia)* source as a logical distinction, or *(ib)* composition as a real distinction; *(ii)* certainty (subjectively) or truth (objectively) as an epistemic distinction; and *(iii)* temporality as a psychological or phenomenological distinction. Moreover, among these respects, Kant uses both 'a priori-a posteriori' and 'pure-empirical' pairs in somehow free and interchangeable fashion. At any rate, this account would still benefit from a proper inventory in order for explicative clarity at least for the remainder of this study.

Accordingly, with respect to their *(i)* origin, our cognitions can have, *(ia)* logically, a priori (that is, they can be derived from our cognitive faculty) and empirical sources (that is, they can be derived from experience); (4:266) but, really, they can only arise only from either the cognitive faculty; or from both the experience and the cognitive faculty, that is, in terms of their *(ib)* composition, they can be either pure or composite. Accordingly, there is no cognition composed *purely* of that which is empirical. With respect to their *(ii)* certainty, cognitions can be established either a priori, that is, their truth or certainty can be established independent of experience; or a posteriori, that is, their truth or certainty can only be confirmed through experience. Finally, in terms of *(iii)* temporality, which is a psychological or phenomenological distinction, all our cognition is a posteriori "as far as time is concerned" (B1) that is, it begins with experience. While the last one may seem at first rather too plain or perfunctory, it is crucial in its negative indication, namely that pure concepts having their seat in our understanding are developed only on the occasion of experience; (A66/B91) and that, without data, even the elements of a priori cognition are not to arise [merely] in thought. (A96)

In this light, 'pure-empirical' pair does not seamlessly overlap with 'a priori-a posteriori.' While pure cognitions are only possible a priori (pure a priori), cognitions that are certain a priori can nonetheless be also mixed in their origin (non-pure a priori and a priori *secundum quid*). While these [*qua* judgments] are both established a priori, only the former is designated properly a priori in Kantian terms [and the latter is not]. Moving forward, empirical cognitions are only possible a posteriori, and conversely; however, while 'a posteriori' [as modifying judgments] can be and are used interchangeably with either 'experiential' or 'empirical,' the terms 'experiential' and 'empirical' should not substitute each other, for the experience is a composite whose form is contributed by our cognitive faculty, *i.e.,* a priori, and whose matter by sensations, *i.e.,* empirically. Now it is true that, there is no *completely* empirical cognition [without any a priori contribution on our part] in Kantian framework, unless the term contextually refers to *(ia)* source [and not what arises [partly] out of such source]; yet when it is used freely as opposed to a priori either in respect of composition *(ib)* or certainty *(ii)* this warning is duly ordained.

2.2. Analytic and synthetic

Kant distinguishes (A6/B10 ff.) between two types of judgments, as based on two different ways wherein the relation (*Verhältnis*) of subject to predicate is possible, namely, analytic and synthetic.

Accordingly, analytic judgments are those where [the concept of] predicate is already contained in [the concept of] subject, even if covertly so. In contrast, synthetic judgments are those where [the concept of] predicate lies entirely outside [the concept of] subject, and therefore not at all contained therein, albeit as still being connected (*verknüpft*). (Ibid.)

Analytic judgments break the subject up into its partial concepts and recognize the predicate there [as a partial concept]. The predicate thus would add nothing but only determine what is already thought, even confusedly, in the subject. Analysis merely sets out and dissects the concept we already have and makes it intelligible (*verständlich*). Following this, analytic judgments can be considered as judgments of clarification (*Erläuterung*), for they do not expand our cognition but only clarify what we already have cognized. (Ibid.)

By contrast, synthetic judgements add to the subject a predicate that was not [readily] thought therein and consequently that could not have been extracted from it by analysis. In this respect, synthetic judgments expand our cognition and on this wise they can be considered as judgments of amplification (*Erweiterung*). (Ibid.)

In both types of judgments, however, the predicate belongs to the subject *ultimately* in that they are connected, for understanding, through such connection, judges that the predicate belongs to the subject. In analytic judgments, this connection is through identity, as the predicate is found "by becoming conscious of the manifold of component concepts that is thought" in the subject and so is their connection. In synthetic judgments, on the other hand, this connection cannot be thought with

identity, as the predicate is something "entirely different" from that which is already thought in the [mere] concept [of subject]. In synthetic judgments, therefore, there must be available to understanding, in addition to the concept of the subject, "something else," *viz.,* some [*sufficient*] *reason*, a "third thing," (A155/B194) on which it would depend "in cognizing a predicate that does not lie in that concept as nevertheless belonging to it." (A6/B10 ff.)

2.3. Classification of judgments

Because all the conditions of the judgment are already contained in the subject, no confirmation or contribution from elsewhere [*e.g.,* experience] except from the concept itself is required for an analytic judgment, which is therefore established a priori [and only a priori]. In rendering an analytic judgment, the predicate is merely drawn out from the subject by analysis in accordance with the principle of contradiction which simultaneously imposes the necessity unto the judgment. "For since the predicate [...] is already thought beforehand in the concept of the subject, it cannot be denied of that subject without contradiction," (4:267) that is, its negation would yield a contradiction. All analytic judgments are therefore a priori judgments [and can never be a posteriori], regardless of whether its concepts are of a priori or empirical origins. (*q.v.* fn. 10) Indeed, "[such a] procedure [of analysis] is no different [...] in any [...] type of cognition when one seeks simply to make its concepts clear through analysis." (4:273) Accordingly, even if its concepts are empirical, an analytic judgment, since the concept is given, is rendered a priori (4:267).

The principle of contradiction is thus "the common principle of all analytic judgments" (4:267) and by whose virtue they are thus established necessarily true a priori. While the principle of contradiction is a necessary condition of all judgments, it is both necessary and sufficient for analytic judgments.

A posteriori judgments [*i.e.,* empirical judgments, or judgments of experience] are all synthetic, for what is predicated therein of subject is not contained in the subject and is given from elsewhere [*i.e.,* [empirically] through experience] and the judgment is thereby established. Although the predicate is not contained in the subject, the subject "designates an object of experience [...] and [accordingly] other parts of this experience can be added," contingently, to the subject. At that, both subject and predicate [as parts] belong together as connected in and through [the whole] of experience. Here, the experience *qua* empirical intuition is that "something else" which grounds the synthesis, *i.e.,* on which understanding depends in synthesizing the predicate with the subject. (A6/B10 ff.)

However, while all a posteriori judgments are synthetic and all analytic judgments are a priori, not all a priori judgments are analytic. There is also a third kind, that is, "synthetic judgments that are a priori certain and that arise [solely] from pure [...] reason," (4:268) judgments wherein the predicate is not contained in the subject and nevertheless established as necessarily true and universally valid. (A6/B10 ff.)

In consideration of the foregoing, three classes of possible judgments are thus analytic a priori, synthetic a posteriori, and synthetic a priori.[11] (*q.v.* §3.3.1)

[11] Kant thereby shifts the axis of early modern epistemology. Now, no philosophical creation stands in vacuum but only in its time [and place]. This is perhaps particularly so in the case of Kant, as his conscious dialogue with his predecessors and peers [and even with the posterity] throughout the *Critique* attests. While such an historical context does not necessarily mean the absolute historicization of the underlying idea, it can still be used as a means to better understand it. Even though such a genealogy can only approximate to a crude schematization, which would be perhaps not wrong but still "woefully incomplete," (Beck, "Analytic and synthetic judgments before Kant," 81) it can be taken for the purposes of an introduction, if not as a serious history of philosophy.

In any case, the problem [of the *Critique*] itself is properly historical and one that is nothing short of domineering the mainstream of early modern philosophy, which is marked [in hindsight] by Kantian synthesis and overcoming (*Aufhebung*) of the orthodox theories of 'human understanding' of rationalism and empiricism. The means to this was none other than the synthetic a priori.

In early modern philosophy, despite all their fundamental differences, both rationalists and empiricists, as epitomized by, *e.g.,* Gottfried Wilhelm Leibniz and David Hume, recognized the long-standing distinction between two fundamental types of knowledge. The first kind, designated as 'truths of reason' by Leibniz and 'relations of ideas' by Hume, were propositions expressing necessary and universal truths which could be established by logical principles [alone], hence a priori. These were discoverable merely by thought without any recourse to experience. The second kind of knowledge, in diametrical opposition to the first one, designated as 'truths of fact' by Leibniz

Mathematical propositions

Kant, with his classification of judgments and the introduction of synthetic a priori, challenges not only the earlier classification which was well-entrenched and curiously ubiquitous in the tradition, but also another established conception that mathematical judgments are a priori [hence] analytic. Conversely, Kant asserts that while mathematical propositions are indeed a priori for they are universal and necessary, they are synthetic for they extend our cognitions.

In order to illustrate this thesis, Kant proceeds with two examples. Accordingly, when we delve into the proposition <7+5=12>[12] we find that the concept of 7+5 contains nothing more than "the unification of the two numbers into one." In other words, we cannot analyse the concept of 7+5 to find the concept of 12 therein. (4:268) In order to find the concept of 12, we "must go beyond these concepts" by

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and 'matters of fact' by Hume, were propositions expressing contingent truths which could not be established merely by logical principles and had to be derived from experience by observation and induction, thus only probable a posteriori.

For Leibniz and Hume, necessary and a priori judgements would translate to analytic, and contingent and a posteriori judgements would translate to synthetic. Kant, too, so far, acknowledged this distinction and agreed with it to the extent that contingent and empirical judgements are indeed synthetic, and analytic judgments are indeed necessary and a priori. However, according to Kant, while all analytic judgments are indeed a priori, not all a priori judgments are analytic. Kant thus proposed a new class of "synthetic judgments that are a priori certain and that arise from pure understanding and reason," (4:268) where that which is predicated is not already contained in the concept of the subject of the judgment, and nevertheless necessarily true and universally valid.

Accordingly. In pre-Kantian thought, [a priori] universality and necessity were conferred only on analytic judgments, which are necessarily true by virtue of the principle of contradiction—hence the earlier classification of [analytic] a priori (~relations of ideas or truths of reason) and [synthetic] a posteriori (~matters of fact or truths of fact) judgments. By maintaining that there are synthetic a priori judgments, Kant also affirms *ipso facto* a necessity [a priori] that does not arise [merely] from identity. Kant thereby does not only simply add a third tine to Hume's fork, but also modifies its shape and form altogether into now what would be called Kant's trident.

N.B. The onto-epistemological character of Kantian hypothesis thereby gives rise to yet another synthesis and overcoming of another mainstream dichotomy of [early modern] philosophy: idealism and realism. Kant is an idealist in that we can only experience the phenomenal world, however he is a realist in that this world is not a [mere] product of our minds but is given to us. "It is the combination of [idealism] and [realism] which produces the thing in itself." (Paton, 68) Space and time as transcendentally ideal and empirically real *forms* of our sensibility only have application to the *matter* of objects that are given to us—hence the *formal* [or critical or transcendental] idealism.

[12] This example is also found in Plato (*Theaetetus*, 196a).

"making use of intuition" and add *successively* the units of 5 [in the intuition] to the concept of 7. (Ibid.) Since the sum cannot be found through a *mere* analysis of concepts [without *also* making use of intuition], [such] an arithmetical proposition is always synthetic. (Ibid.)

As for geometrical propositions, when we likewise delve into, *e.g.,* the proposition <the straight line between two points is the shortest,> we see that the concept of straight line does not imply [*i.e.,* contain] anything with regard to length. The concept of the shortest is therefore an addition which could not be extracted through analysis from the concept of the straight line. Consequently, [such] geometric propositions are likewise synthetic.

Basically, syntheticity of mathematical propositions signifies that a mathematical proposition is both necessary and amplifying, simultaneously. It is a priori for it is necessarily true, and it is synthetic for it amplifies our cognition. The explanation for this *seems* to be conceptual containment. Be that as it may, the argument for the syntheticity of mathematical propositions is not [merely] limited thereto. Indeed, conceptual containment, if it is actually the case, would only be the effect and not the cause. Or, more accurately, the conceptual containment is due to that mathematical cognition proceeds "not from [mere] concepts, but always and only through the construction of concepts." (*q.v.* §4)

2.4. Formulation of syntheticity

Although Kant attends only briefly to the direct treatment of analytic and synthetic judgments in the *Introduction* (A6/B10 ff.), the demonstration of their possibility is dispersed through the *Critique*. Accordingly, further explanations or questions regarding the distinction between analytic and synthetic judgments are implicit as to be found therein.

Now, to enumerate, provisionally, the criteria as to distinguish between an analytic and synthetic judgment as per the explicit formulation are as follows: *(i)* whether predicate is *contained* in the subject; *(ii)* whether predicate is *actually thought* in the subject; *(iii)* whether predicate is *connected* to the subject by the *principle of identity*, or alternatively, whether the judgment is established by the *principle of contradiction* [alone]; and finally *(iv)* whether predicate *adds* to the [content] of the subject, or alternatively, whether the judgment *augments* our cognition. It can be argued that, whereas *(i)* and *(iii)* appeal to logical respects, *(iv)* appeals to epistemological respects. Nonetheless, it can also be inferred that these are inseparable, and consequently, they should be nothing but manifestations of one and same criterion.

Still, the distinction is primarily articulated in terms of conceptual containment and, taken as such, this account would only serve to defer and displace the issue. That is, the question, *e.g.,* as to whether a judgment is actually analytic or synthetic, would only be alternatively expressed as the question, *e.g.,* as to whether a predicate is actually contained in the subject. In this regard, for a fuller insight into this philosopheme, it is only sensible to consider the explicit formulation of the distinction against its theoretical background, to identify its bearings thereunto and systematically reconstruct these into a working framework. At any rate, "whatever be the difficulties of the distinction, it can hardly be denied that we make synthetic judgments and some of these appear to be a priori. [If] so, Kant has a real (*wirklichen*) problem."[13]

[13] Paton, 87.

3. GENERAL FRAMEWORK

Its concise formulation (*q.v.* A6/B10 ff.) notwithstanding,[14] the analytic–synthetic distinction [of judgments], which "appears to come forward of itself," (4:270) is [rationally] underwritten by, hence comes forward from, a wider logical and metaphysical doctrine, which, taken together, would comprise [what is in *modern* terms called] *Kant's theory of knowledge*. Accordingly, how this distinction, as a philosophical design, is adequately grounded by and what theoretical tenets and how its [explicit] formulation, as a technical design, adequately upholds the [underlying] idea of a synthetic a priori cognition as addressing the critical problem are to be seen in this light.[15]

Since cognition is the pedestal [logico-ontological] unit at which the distinction is essentially directed, this prospective [general] framework for syntheticity is first and foremost a framework of cognition, which follows from the mechanics of logic and metaphysics. Within this context, logic governs the mere form of discursive cognition, *i.e.,* interrelation (*Verhältnis*) of cognitions [to one another], wherever these may originate; whereas metaphysics pertains specifically to a priori cognition of reason and of its objects that originate a priori. (*See also* fns. 25, 37)

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[14] *Cf.* "A reviewer who wanted to say something censuring this work hit the mark better than he himself may have intended when he said that no new principle of morality is set forth in it but only a new *formula* [...] But whoever knows what a formula means to a mathematician, which determines quite precisely what is to be done to solve a problem and does not let him miss it, will not take a formula [...] as something that is insignificant and can be dispensed with." (5:8, emphasis added)

[15] This *rational* exigency for further *logical distinctness* in elaborating syntheticity to a technical extent sufficient for the ends hereof, would also be complemented by an incentive for furthering *historical* affinity therewith that offers an illuminating *aesthetical distinctness* admissibly no less legitimate if not perhaps as pressing. For indeed, terms of the framework Kant had worked out over the years in the *marginalia* (*q.v.* §1) in the form of a criticism (*q.v.* fn. 8) is largely inherited from the tradition. Assuredly, this continuity (*see, e.g.,* Bxxxvi) was by no means alien to Kant's contemporaries as it might be now alas to the modern inquirer who might lack sufficient background in the 18th century logic and metaphysics of the rationalist Leibnizian–Wolffian school as thoroughly studied and taught by Kant (*q.v.* fn. 1b). This is perhaps all the more warranted, considering the terminology which for the most part had been already clarified, refined, and, above all, systematised by this genealogy; as well as the particular methodology and mannerisms which were typical of the school.

3.1. Cognition in general

To begin with that which is the "simplest in our cognition," (24:904) representation (*Vorstellung*) is the fabric of our entire cognitive framework, that is, the most basic element that constitutes all our cognition.[16] While it is beyond exhaustive definition[17] and laden with connotations,[18] [a] representation [as such] signifies a determination in us [*qua* the subject] as a modification of our mind (A97), which relates (*beziehet*) to something [*qua* the object], in that "the former, as it were, substituting (*vertritt*) in us for the latter." (11:395) Consequently, there is nothing

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[16a] *Vorstellung*, *Vertretung*, and *Repräsentierung* are terms all used by Kant and are all likewise translated as representation. "Whereas Kant uses the terms *Vertretung* and *Repräsentierung* to designate especially the action or relation of representation, he uses the term *Vorstellung* to designate a mental state, a determination in us that has the value of representation." (Cassin, 891–2).

[16b] In Kantian scholarship, it is also not uncommon to render *Vorstellung* [instead] as 'presentation' at the philosophical discretion, in view of that "Kant's theory of perception is not representational." Werner S. Pluhar, a prominent 'representative' of this position, distinguishes between the nuances of the term and asserts that "*vorstellen*, in the Kantian use of the term [...] is not something that *Vorstellungen* do; it is something that we do;" and identify 'presentations,' as "objects of our direct awareness as sensations, intuitions, perceptions, concepts, cognitions, ideas, and schemata." *See* Pluhar tr. (22); *see also* fn. 18.

[17] In being the most basic element, the term representation is inevitably self-referential, as Kant underlines several times in his logic lectures (*e.g.,* 9:34; 24:40; 24:701, 730, 752; 24:805). It is a fundamental simple concept that we necessarily must have, (24:40) which can neither be [further] analysed nor be defined, for it would always take yet another representation to explain what it is.

[18] Traditionally, the term [representation] may designate "*(i)* a [simple] relation[ship], *(ii)* an [relational] action of representing, *(iii)* a vehicle of representation." This polysemy, this "open texture" of representation, which tests and stretches the ontological status of the term and from which German is not exempt, is also fairly present in Kantian corpus.

Ordinarily, a representation is typically *of* something, thus intentional. Nevertheless, wherever *X* represents *F* the term may indicate that *X,* the representation, is the vehicle of representing *F*; or that *X*, the agent of the action of representation, represents *F* to itself. Similarly, < representation of *Y* > may refer either to the vehicle or action of representation of which *Y* is that which is represented; or, again, to *Y,* as that which represents as the agent of the action, *etc*. When taken to be a vehicle, representation emerges as an entity that is "external to the relationship of representation in the sense in which [it] can be characterized independently of the fact that it enters into a relationship of representation" and that may lead to an "abusive reification" even would this not have been the case. (Cassin, *loc. cit.*)

In Kant, all of the foregoing senses are applicable as per the context. Kant's usage of the term *Vorstellung* is "not only compatible with the semantic profile of the [...] term 'representation' but responds explicitly to the main questions left open by [it] [...] Still this can give rise to a certain intellectual tension [in that] either Kant's *Vorstellung* has an intrinsic value of representation, in which case it would be wrong to reify it, or it is only a vehicle of the representation and can be characterized independently, though evidently Kant does not do so." (Ibid.) This vehicle–action ambiguity [with respect to representations] also extends [especially] to cognitions and intuitions [which are also representations (*inf.*)]. On this, *see also* Paton (94 f.).

in us that is not a representation or that we do not have represented. (9:33; 24:40; 24:701 f., 752; 24:805, 904; A320/B376)

Much as cognition is defined both in reference to and in terms of representation, they are not strictly one and the same.[19] That is to say, while cognition is a [qualified] representation, representation [as such] is not yet cognition. In order for a representation to qualify as cognition, it must first be combined with consciousness.[20] (Ibid.)

When we distinguish a representation and its object from other representations, that is, when the [degree of] consciousness is at least sufficient for differentiating a representation from the others, then we are conscious of that representation and [accordingly] that representation is clear. Such a representation with consciousness, a clear representation, is called perception. Thus, there is nothing *for us* that is not a perception or that we do not have perceived. (Ibid.)

Again, not all perception, as such, is yet cognition. Perception is called sensation in its relation (*Beziehung*) to the subject as a modification of his state,[21] and cognition in its relation to the object.[22] Representation, then, qualifies as cognition only

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[19] However, cognitions and representations can be treated in logic as synonyms (24:70) "without committing an appreciable error" (16:78). It is largely done so in Kant's *œuvre,* including the *Critique,* which generally concerns neither obscure representations nor mere sensations.

[20] Although an unconscious representation is as good as nothing *for us*, [the concept of] representation [as such] is necessarily presupposed (24:730) in [explaining] cognition. In this view, it would be more befitting to consider representation as defined in terms of and in reference to cognition, rather than the other way around. *See also* fn. 19.

[21] Sensation [as yet] is a mere representation that is not distinguished by a particular object. Hence through [mere] sensation one does not *cognize* an object. So, sensations [as such] are not cognitions. "They [lie] merely in [the subject] and not in the object." (24:701 f.)

[22] In view of that "[*Beziehung*] is the categorial term which deal with perspectives," this qualifier should better be read as '[in] perspectival relation' (Palmquist, 43) or as 'relative [to].'

The German terms *Beziehung*, *Verhältnis*, and *Relation* are commonly rendered in English as relation. Alas, this indifference points to "an unfortunate equivocation" for the distinction is important and very much informative. "[*Relation*] ordinarily refers [...] to the category of relation or to its application in specific cases [in which 'relation' is obviously correct] [...] [*Verhältnis*] is almost always used, like *Relation*, in some categorial sense, usually as a reference to a relation between two or more items of the same or similar type [*i.e.*, in a sense closer to 'interrelation'] [...]

insofar as it is related with consciousness to its object, or what is the same, insofar as the object is attended to in a perception. (Ibid.)

Cognition relates to objects immediately as intuitions (intuitive cognition) and mediately as concepts (discursive cognition). (24:40; 24:805) An intuition is a singular representation (*repraesentatio singularis*) of the individual [object] to which it relates immediately. A concept is a [reflected] representation (*repraesentatio discursiva*) of that which is universal (*allgemein*) or common (*gemein*) to several objects to which it thereby (*per notas communes*) relates mediately. Alternatively, intuition is a single representation while concept is a [common] representation insofar as it is contained *in* various other representations, (A320/B377; 9:91: 24:701, 752; 24:805 f., 904) or what is the same, insofar as it contains other representations *under* itself.

Now, although intuitions and concepts are each, to be sure, conscious objective representations, neither intuitions nor concepts, alone, yield cognition. Cognition is more than mere concept and mere intuition—it is their combination. Therefore, it is necessary to make both concepts sensible and intuitions intelligible (A51/B75); that is, without appending the intuition to the concept, *i.e.,* exhibition (*Darstellung*), there can be no cognition whatsoever.[23] (20:325; 24:752)

"*Beziehung* [...] by contrast [to *Verhältnis* [and *Relation*]] almost always implies a more abstract, perspectival relation [...] in which one side [...] establishes a perspective for viewing the other side." Particularly, *Beziehung* in this sense is very characteristic for both Kant's philosophizing and philosophy and is employed commonly. On this, *see* Palmquist (43–6).

[23a] Consequently, one can identify two discernible senses of cognition in Kant's theoretical philosophy. In broad sense, cognition is objective conscious representation which, in turn, can be *either* intuitive (intuition) *or* discursive (concept). However, in narrow sense, cognition arises only from the amalgamation of concepts *and* intuitions. While it is evident that it is the latter sense, as one of the foundational tenets of the critical philosophy, that prevails; there are various expressions dispersed through Kantian corpus, which, when taken out of context, may seem to imply otherwise: *e.g.,* that one cognizes concepts or intuitions (*e.g.,* 24:274); that one cognizes through concepts or intuitions (*e.g.,* 24:845); that there are two sorts of cognition (*e.g.,* A294/B351); that one's cognition involves intuitions and concepts (*e.g.,* 24:705); that cognitions are [either] intuitions or concepts (*vel intuitus vel conceptus*) *etc*. Kant addresses the confusion specifically in the *Progress in Metaphysics*, maintaining that whereas cognition requires both the intuitive and the discursive element together, "it is [only] *called after* that to which [one] particularly attend[s] on each occasion, as the determining ground [of such cognition]." (20:325, emphasis added)

Our cognition thus has two sources, from which intuitions and concepts, respectively, arise: sensibility and understanding. This dual structure brings about two aspects of cognition as real (material or transcendental) and logical (formal), imparting to cognition both metaphysical and logical characteristics.[24] The science which deals with the [formal] use of the understanding is logic, which discusses the [universal] objects of the understanding metaphysics. (24:31)

3.2. Logical framework[25]

Sensibility, the lower faculty of cognition, supplies the material [for the understanding]; understanding, the higher faculty of cognition, brings it under concepts *qua* rules. Human cognition [much as it is sensible] is thus *ultimately* discursive. (9:58; A68/B93)

Technically, cognition has a relation to the object and to the subject. In the former respect it concerns representation [of the object] *qua* concept; in the latter, consciousness [of this representation] *qua* distinctness [of the concept]. (9:33 f.)

[23b] At that, there is a further nuance that should be noted adding up to the polyvocality of the term, between [the act of] cogniz-ing and [the vehicle of] cognit-ion as corresponding respectively to the [logical forms of] judgment and concept. (*q.v.* §§ 3.2, 3.2.1; *cf.* fn. 18)

[24] A43/B61–2: "The Leibnizian-Wolffian philosophy [...] directed all investigations of the nature and origin of our cognitions to an entirely unjust point of view in considering the distinction between sensibility and the intellectual as merely logical, since it is obviously transcendental, and does not concern merely the form of distinctness or indistinctness, but its origin and content." *See also* fn. 41.

[25] [General and pure] logic is a *canon* of the understanding in its formal use, that is, it contains the [absolutely] necessary rules of thinking [*i.e.,* discursive cognition] without which no use of the understanding takes place, regardless of the difference of objects to which it may be directed. Logic thus abstracts from all content of cognition, that is, from any relation (*Beziehung*) of it to the object and considers only the form of thinking in general, that is, the relation (*Verhältnis*) of cognitions to one another. (9:11 ff.; A52–5/B76–9)

Logical procedure of cognition in objective respect

Stated in logical terms, the discursive unit of cognition is *called* a mark (*nota*). A mark is that [in a thing] which constitutes part of its cognition (*cognitio partialis*). It therefore belongs as a partial representation to the whole [possible] representation [of this thing] *qua* its ground of cognition. Concepts relates to intuitions through certain marks. (A19/B33) Marks—cognized together—give a positive [whole] representation, *i.e.,* the concept, of the thing.

Marks can be intuitive, that is, they belong to intuition as its [synthetic] parts; or, discursive, that is, they belong to concept as its [analytic] parts.[26] Intuitions are [in principle] comprised of [indefinitely] inexhaustible marks which, *in toto*, comprise [one single] mass; wherefore [intuitions] are singular [and not discrete]. All marks inasmuch as they are abstracted from intuitions [and employed in definitions (*inf.*) [of concepts] *qua* ground of cognition] are [partial] concepts, since once they are abstracted and [now] discrete, they are applicable [be it potentially] to several things and not exclusively to one particular individual.

[26a] All the same, be it intuitive or discursive, a mark nevertheless is a concept. That is to mean, all marks are concepts; (9:58; R2286; *cf.* 20:325) but while not all concepts are [necessarily] marks, all empirical concepts are marks [of intuitions].

[26b] Perspectival relations (*Beziehungen*) and 'contextual convertibility,' (*q.v.* fn. 22) may seem to bring about an inflation of terminology [for the novice], which [does not] [but which] in any case, should perhaps be combed out to avoid confusion. Put in this context, all marks are representations [much like anything we cognize], but not all representations are marks [*i.e.*, of things]: *e.g.*, feelings, [mere] sensations. A mark is that which is [at least potentially] common to many things (*nota communis*) and not exclusive to an individual thing (*nota singularis*). All marks are therefore [partial] concepts *qua* ground of cognition [of [the whole] concept]; or what is the same, all concepts are marks insofar as they are contained under other concepts. However, the proper use refers to mark as [residing] in the *thing*, otherwise its use would be redundant, for the term partial concept would already have made a perfectly sufficient terminology. Likewise, not all concepts, too, can be considered as marks: *e.g.,* [unconditioned] ideas. At any rate, Kant uses marks, representations, and concepts interchangeably and this is surely fine [in most contexts], because they are almost always interchangeable save for the [possibly] exceptional cases noted.

Consequently, intuitions [*qua* ob–jects] are always already thoroughly determinate [and singular]; concepts [*qua* objects] are always abstract [and common].[27] Logic does know but only two fundamental operations: abstraction and determination. Through logical abstraction higher concepts arise and through logical determination, lower concepts. However, a concept is never thoroughly determinate, *i.e.,* as that which no further determination might be added in thought, but always determinable, (9:99; 28:552) for there is no lowest species [of concepts] which can match up the particular individual [which can only be an intuition].[28] Concepts and intuitions therefore constitute two [reciprocal] endpoints of human cognition.

The individual is the [real or antecedent] *determining* ground of the concept (*q.v.* fn. 42). Conditions of an [empirical] concept hence lies in sensibility. Concepts thus relate to objects [mediately] through intuitions, and to intuitions through certain particular marks [that it abstracts therefrom] and thereby constitute a particular [concept of] object [consisting] of those particular marks. It is through marks the understanding dissolves intuitions and puts them together [as concepts [of objects]]. Reason thus goes from that which the marks contain to that which the whole

[27] Accordingly, the expression 'abstract concept' makes a redundant signification, for all [empirical] concepts are abstracted from different marks in [concrete] individuals [*qua* intuitions] and, in this respect, they are thus all *abstract-ing*. However, concepts can be used [relatively] *in abstracto* or *in concreto* depending on the level of their determinacy. For instance, the most abstract concept is "that which has nothing in common with anything distinct from itself," (9:95), *i.e.*, the concept of 'something' which is different only from [the concept of] 'nothing' with which it has nothing common. (*Cf.* 28:543 ff.; 29:811, 29:960 f.) There is, however, no most determinate, *i.e.,* singular, concept. (*sup*.)

[28] Since there is no singular [or 'complete'] concept [but only a singular use] in Kantian logic, the problem of singular reference, that is, how individuals are predicated in a judgment which admits only concepts and not intuitions, is a matter of scholarly controversy. Nevertheless, *lex parsimoniae* permits, at least for the purposes of this study, that Kantian notion of *whole* concept [or representation], albeit perhaps not expressly so theorized, adequately substitutes for Leibnizian 'complete concept.' Assuredly, a whole representation may nevertheless be clear *totaliter* despite being obscure *partialiter*, and thereby can amply refer to the individual *qua* whole representation: *e.g.,* the concept of Socrates need not be thoroughly determinate to relate [and refer] to the individual Socrates. Still, the problem happens to be merely displaced into another—and the one which would now concern the context hereof. That is, how *essentialia* of the clear but indistinct whole concept is peremptorily determined without [all] marks being enumerated, and how then there can be a *whole* concept without an *essentialia*? (*inf.*)

concept contains. In light of this, to cognize [things] comes *precisely* from to be acquainted [with marks]. (9:58; R2281; *see also* Diagram 2). Through intuition marks [of the ob–ject] are acquired (*erworben*), or what is the same, we are thereby acquainted (*kennen*) with the ob–ject; through concept, it is conceived (*konzipieren*), or what is the same, we thereby have the [corresponding] object cognized (*erkennen*).

Now, a mark belongs as a partial representation to the whole [possible] representation [of this thing] *qua* its ground of cognition, whereas this ground of cognition is of twofold use: it serves internally, for derivation, to cognize the thing; or merely externally, for comparison, to differentiate it from other things. Marks through which the thing [to which these marks belong] can be *always* distinguished are sufficient. (9:58; R2283; R2270; R2280)

Therefore, it is sufficient marks, cognized together, that gives a positive [whole] representation, *i.e.,* concept, of the thing. Those which must *always* be found in the thing represented [so that it could be that thing] are essential marks. Among essential marks, those which belong to the thing as grounds of its other marks are constitutive marks (*essentialia in sensu strictissimo*), while those others which are derived from the constitutive marks as their consequences are attributes. Those which can be separate from the concept of the thing are extra-essential and accidental marks. They concern either internal determinations of the thing (*modi*) or its external relations (*relationes*). The sum (*Inbegriff*) of *essentialia* is the essence.[29] (9:60-61; 24:834)

[29] Essence may refer to logical or real essence (*vel logica vel realis*). Logical essence (*esse conceptus*) [of the concept] is the ground of all [logical] *predicates* of a thing, *i.e.*, of all that which is contained in [its] concept; and includes nothing further than cognition of all necessary predicates in regard to which an object is determined through its concept. Real essence (*esse rei*) [of the thing] is the ground of all *determinations* of the thing, *i.e.*, of all that which belongs to the matter itself; and it requires cognition of those predicates on which everything that belongs to the existence of the thing depends. Logical essence is found through analysis whereas real essence is arrived at through synthesis. We can have insight into logical essence, yet into real essence we never can. (*Recited from* 28:552–3; 9:61)

Logical procedure of cognition in subjective respect

Since cognition is the representation [of an object to which it is related] with consciousness, (*q.v.* §3.1) the form of all cognition (*inf.*) depends on consciousness which is manifested in clarity [and distinctness (*inf.*)] of representations.

The consciousness of a representation that is sufficient for distinguishing it [from another] is clarity. So far as a representation is [at least] differentiated, it is clear; otherwise, it is obscure. Clarity is therefore the *conditio sine qua non* for any cognition. However, [mere] clarity is not always altogether a sufficient [condition].

Representations, in terms of their quantity, *i.e.,* multitude and manifoldness, (9:40) can be either simple or complex. A simple representation is [*in*] itself a single mark [as a ground for cognizing either its own or other [complex] representations of which it is a part, as the case may be], hence contains no manifold. A complex representation, on the other hand, consists of multiple representations, and thus contains likewise a multitude of marks. While [mere] clarity is sufficient to both [externally] differentiate and [internally] derive simple representations; it is only sufficient to differentiate complex representations [externally] but not to derive these [internally]. Now, if one is conscious of the [whole] representation, *totaliter*, as to its difference from other representations, but not, *partialiter*, of the manifold [of marks] therein, then such representation would be [still clear but] indistinct [or, confused].[30]

A second stage, then, a higher degree of clarity, is distinctness, *viz.*, the consciousness whereby [marks of] complex representations become clear *partialiter*.[31] Distinctness is thus clarity that extends to the parts, in accordance with

[30] "What is confused must be [complex], for in what is simple there is neither order nor confusion. Confusion is thus the cause of indistinctness [not the definition of it]." (7:138)

[31] Distinctness [like clarity] is both an aesthetical and a logical *quality*: the former, distinctness of intuition, is consciousness of the manifold contained in the intuition; the latter, distinctness of concept, is development of cognition [as to form] by developing partial representations. (9:35)

which all clear [complex] representations are logically further distinguished into. A cognition, then, is distinct insofar as it has clear marks. (24:834 f.)

Now, every cognition, since it requires intuition and concept together, is inherently a complex representation [containing multitude of marks]. Therefore, through distinctness alone a complex of representations can become, *i.e.,* combined into, cognition, whereby order is thought in this manifold, as every combination presupposes a rule [for order] [*i.e.,* a concept.]

Distinctness [of concepts] is attained by the ordering [of marks] according to which they are combined, through either a [merely logical] division into higher and lower marks (*primaria et secundaria*) or a [real] division into principal and accessory marks (*principalis et adhaerens*). (7:137 f.; 9:33 ff., 61; 24:702; 24:805 f.)

Marks are ordered [as to their sequence] *after* or *under* one another. They are *coordinate* insofar as each mark is an immediate mark of the thing (*nota proxima*) and *subordinate* insofar as one mark is represented only by means of the other (*nota remota*). The combination of coordinate marks as to form [the whole [of]] concept is an *aggregate*, that of subordinate concepts is a *series*. Aggregation of coordinate marks constitutes the totality of the concept. The series terminates *a parte ante*, on the side of grounds, in simple concepts *qua* the highest *genus*; *a parte post*, on the side of consequences, for there is no lowest species. (9:59–60)

Marks are [so] ordered *in* concepts and *by* judgments, Distinct concepts can only arise through judgments. "The concept to be made distinct through comparison with its mark, is the subject, and that concept which is added to the mark as ground of distinctness is the predicate." (R3046; 24:273 f.)

To cognize [an object] is then to cognize [it through] a distinct concept [of the object]. The distinctness of cognitions depends on the distinctness of concepts as to both their content and extension. Every concept as a partial concept is contained *in*

the representation of things, and to that extent has a content; and, as ground of cognition, these things are contained *under* it, and to that extent it has an extension. The former is furthered through definition while the latter through [logical] division.[32] (9:59, 95, 140)

Distinctness is attained logically by analysis of marks (*per analysin*) or by synthesis of marks (*per synthesin*), which generates, respectively, analytic and synthetic distinctness.[33] Definitions of concepts that are given (*conceptus dati*), whether a

[32a] Definition is distinctness through sufficient marks (*completudo*) and *only* sufficient marks (*praecisio*). A concept that is adequate in minimal terms, that is, a complete concept determined precisely [and originally], is a concept adequate to the thing (*conceptus rei adaequatus*). (9:140; 24:756) To define [in proper sense] (*Definition*) means to exhibit originally (as not derived from anywhere and thus in need of a proof) the exhaustive concept of a thing (sufficiency) within its boundaries (precision). *(i)* Concepts given a posteriori [*i.e.,* empirical concepts] cannot be defined but only explicated (*Explikation*), for these are [provisionally] comprised of marks abstracted from [corresponding] indefinitely determinate ob–jects and their boundaries always change subject to [new] observations, therefore are never precise. Totality of [synthetic] empirical concepts, can never be completed, but rather resembles a straight line without limits (*Grenze*). (9:59) Consequently, such an explication merely designates (*bezeichnen*) [and not constitutes the exhaustive] concept of the thing. *(ii)* No concept given a priori can be defined either, for they *may* [always] contain obscure representations, making the exhaustiveness, therefore the sufficiency of their analysis [always] doubtful. So, these concepts may rather be exposed (*Exposition*), and their exposition would be valid to a certain degree [as to explain (*erklären*) the concept] but not apodictically certain. *(iii)* Concepts that are made can always be defined. For indeed, it is through definition they are made (*machen*). *(iiia)* For those concepts arbitrarily made up, which were given neither through the understanding nor experience, insofar as they do not [yet] correspond to a possible ob–ject, such an explanation is [better] called declaration (*Deklaration*) rather than a definition of an ob–ject. *(iiib)* Only mathematics has definitions. Mathematics exhibits its objects a priori in intuition and thus originally gives its concepts (*Konstruktion*). (A727–30/B755–8)

[32b] Logical division is the determination of a concept in regard to everything possible contained under it, in so far as these are distinct from one another. The higher concept is called the divided concept (*divisus*), the lower concepts the members of the division (*membra divientia*). A division into two members is called dichotomy; that into more than two members is called polytomy. Dichotomy is thus *a priori* [in the form of X and non-X] and can be merely logical. Polytomy requires cognition of the ob–ject. Dichotomy depends only on the principle of contradiction, however without being acquainted (*kennen*) [as to content] with the concept to be so divide. However, polytomy requires intuition. (9:146 f.)

N.B. "To take apart (*theilen*) a concept and to divide (*eintheilen*) it are quite different. In taking a concept apart I see what is contained in it (through analysis), in dividing it I consider what is contained under it. Here I divide the sphere of the concept, not the concept itself." (Ibid.)

[33a] Analytic distinctness rests on analysis of a given concept, whereby nothing is added but only raised to distinctness. Analytic distinctness concerns those [analytic] marks that are already thought in the concept, *i.e.,* partial concepts of [actual] concept. In analytic definitions the parts precede the whole. When a *concept is made distinct* through mere analysis, cognition does not grow at all as to content. The content remains same and only the form is altered. To analysis pertain thus *making distinct of concepts*. (9:59, 63-4; 24:730)

priori (*rationati*) or a posteriori (*empirici*), are analytic; whereas those of concepts that are made (*ex datis a priori* or *ex datis a posteriori*) are synthetic.[34] (9:59, 140-1; 24:756)

Logical forms of cognition

To cognize through concepts is thought and the [logical] form of all cognition is divided into concepts, judgments, and inferences as per three classical forms of thought (*operationes mentis*). (A133/B171 f.; 9:4, 33 f., 91; 24:701, 763; 24:805, 904). Following this schema, concepts are simple cognitions (*apprehensio simplex*); [simple] cognitions *qua* concepts are combined in judgements (*judicium*); and, finally, combined cognitions *qua* judgements are re-combined in inferences (*ratiocinium*). (24:701, 762–3; 24:904)

As consciousness is the mere form of all cognition, hence all logical forms of cognition depend on consciousness. (9:33; 24:805; B427) Accordingly, [clear] concepts arise through consciousness of universality of concepts (*repraesentatio communis*); distinct concepts [only] arise through judgments whose predicate is the distinguishing mark; [distinct] concepts of reason arise through inference [of reason], deriving a *nota remota* through a *nota media*. (24:701, 762–3; 24:904)

Now, the basic *unit* of cognition is concept. Concept is the object [as that which corresponds to the ob–ject in intuition] inasmuch as it is subject in a judgment; and mark [of the ob–ject] inasmuch as it is the predicate through which the subject is

[33b] Synthetic distinctness rests on synthesis of a [made] concept. Synthetic distinctness concerns those [synthetic] marks of the [merely possible] whole concept that is to come to be through a by adding these. Synthesis begins with the parts and proceeds toward the whole. Marks are acquired (*erhalte*) through synthesis. Thus, a *distinct concept is made* where previously there was none. Synthetic definitions extend cognition as to content through what is added as a mark beyond the concept in intuition. To synthesis thus pertains the *making distinct of objects*. (Ibid.)

[34] Although analytic and synthetic definitions are of Leibnizian–Wolffian legacy from pre-critical Kant (*q.v.* §1), they retain their semantic value for mature Kant. The problematic of this paper could perhaps in abler hands be expressed as the transition from analytic and synthetic definitions to analytic and synthetic judgments.

determined in that judgment, *i.e.,* cognized by being made distinct thereby, as belonging under it [or, what is the same, as containing it].

However, since a concept is composed of [partial] concepts [and partial concepts of further partial concepts and so on *ad in[de]finitum*] "the understanding can make no use of [...] concepts [other] than to judge by means of them." (A68/B93) For indeed, it is only in a judgment two [such] concepts can come to be [[so] related]. That is to mean, a concept is [already] related to other concepts *potentialiter*, while the actual relation of a concept to others as a means for their cognition amounts [precisely] to the [form] judgment. (R3045)

A judgment, as [the representation of] the unity of the consciousness of various representations, or the representation of their relation (9:101), functions as the unity of the relation of the ground [of cognition] to the cognition [of object] in that it represents one concept as contained in another or as excluded from it (Ibid.) according to three relations (*Verhältnis*): subject to predicate, ground to consequence, and the dividing [members] to the divided [concept]. (24:703).

Therefore, we *properly* cognize only through a judgment; or, alternatively, cognition *proper* comes through, that is, is expressed by or incorporated in the logical form of judgment. (A5/B9 f., A78/B103) Prior to judgment, our representations [either *qua* concepts or *qua* intuitions, as such] are nothing but materials for cognition proper [*qua* judgments]. (R4634; *cf.* §3.1, fn. 23)

According to Kant, all functions of the understanding can be traced to judgments, hence, ultimately to these three relations. (A69/B94)

3.2.1. Remarks on logical bearings of syntheticity

Although Kant expressly states that logic "need not even know [the] name" of synthetic judgments, (A154/B192) this is solely because it "has nothing to do with

the *explanation* of [their] *possibility*" (Ibid., emphases added); and at that, rightly so (*q.v.* fn. 45). Nonetheless, the adequacy of [their] formulation, if not the explanation of [their] possibility, would be expected to legitimately concern logic. Granted, it is not a logical distinction; still, it is a distinction [inescapably] articulated in logical terms.

Now, logical framework pertains to [mere] discursive aspect of our cognition; and since our cognition is *ultimately* discursive, all cognition [for us] is *ultimately* subject to this framework [be that only negatively].[35] Likewise, what is formal in our discursive cognitions are expressed in elements of logic. Hence, to that extent, considerations arising therefrom would apply universally [that is, to the analytic–synthetic distinction as well].

The fundamental unit of logical framework is concept, and it knows only [one relation which is logical] containment. The concept device is accordingly limited in what [sorts of marks] it can conceive and how. Concepts, as such, cannot contain real relations but only logical. Essential marks are logically contained *in* the concept, as they determine the logical concept of the object, hence they are logically necessary [for the possibility of concept]. Consequently, they are predicates that can be cognized analytically. Extra-essential marks, however, do not determine the concept [of the object] as such and they are logically extra-essential. That is to say, they are contingent in respect of the [possibility] of logical concept of the object. Hence, they are predicated through judgments synthetically. The highest genus of universal concept framework is the general concept of object, and all concepts are contained thereunder.[36]

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[35] Consequently, the logical framework in a sense underlies the metaphysical model [*i.e.,* transcendental logic] albeit to the extent it is a propaedeutic to [transcendental] philosophy [as well] and canon of the understanding. (*cf.* §3.4)

[36] However, logical framework extends no further than *mere* concepts, that is, it does not reach to ob–jects or recognize what is real in these concepts. The restrictions of such logical framework results with Transcendental Logic, which is a logico-ontological enterprise. So, there the concept framework bifurcates, and one has to concede to a logical and a real plane of concepts, severally. This bifurcation further complicates the issue in respect of containment. (*q.v.* §3.4)

The ensuing logical topography [of concepts] is static and monadic. Kant overcomes the limitations of this traditional framework by assigning [the cognitive] primacy to judgments [*versus* concepts], imparting to cognition a relational characteristic. Cognition is thereby a relation [of partial concepts to each other in a judgment]. While what is cognized is the [sensible] concept [of the object], this cognition comes through [only] in [the logical form of] a judgment.

3.3. Metaphysical framework[37]

Sensibility [*qua* faculty of receptivity] receives representations; understanding [*qua* faculty of spontaneity] cognizes [objects] by means of these representations (faculty of spontaneity). That is, through the intuition ob–ject is *given* to us; through the concept it is *thought*. (A50/B74; 9:35–6)

Still, whereas ob–jects are given to us [in intuition], they are not as such given to us [yet] as objects. To begin with, [some] thing (*Ding*) affects our sensibility and effectuates sensations [*qua* representations] in us. The manifold of sensation [*qua* matter] is ordered in intuition according to the formal conditions of sensibility, *i.e.,* intuited; thereby *appearing* to us [*qua* appearance (*Erscheinung*)] therein as ordered in time and space [*qua* form]. Appearance is thus the sole ob–ject (*Gegenstand*) that we are [ever] given and that which our cognition corresponds to in intuition. [In ordinary sense] the only object (*Objekt*) *versus* us [*qua* the subject] is therefore appearance [and never that thing in itself (*Ding an sich)*].

[37] Metaphysics [in general] is the science of the first principles (*Prinzip*) of cognition. It is of two parts. The first part (*metaphysicam puram*) deals with possibility, domain, and limits of a priori cognition and constitutes a *critique* of pure reason; hence considers not objects [of reason] but reason itself [for its object]. The second part (*metaphysica applicata*), which is the metaphysics in the strict sense, deals with a priori cognition of objects and thus constitutes a *system* of pure reason. In the first part, which is also called transcendental philosophy, reason determines nothing but demarcates only its own faculty of a priori cognition; in the second part, which is also called pure philosophy, metaphysics proper makes use of this faculty. Transcendental philosophy is thus propaedeutic to metaphysics proper. Following the division of human cognition into the sensible and intelligible (*q.v.* §§3.1, 3.2.1), transcendental philosophy is divided into two parts. Transcendental aesthetic is the science of [a priori representations and] principles of a priori sensibility. Transcendental logic is the science of [pure a priori thinking or pure form of understanding] principles of pure understanding in general. (*Cited from* 29:749-53, 756, 759, 803; A21/B33; A52/B76)

The ordered manifold in intuition that is appearance, is further articulated in the understanding according to [the unity of] a priori concepts, *i.e.,* the categories, and synthesized thereby as an object (*Objekt*) [of thought] *qua* phenomenon (*Phänomen*).

The object is [conceived] in the [form of] concept, and that which corresponds to this concept [of the object] is the ob–ject in intuition. To cognize [an object] thus is to have formed the [concept of] object. Objects, in this sense, are concepts; whereas [sensible] concepts [that have objective reality] has [their] ob–jects.[38]

Concepts relate to intuitions and representations thereby become cognitions in the unity of [a priori] synthesis of the manifold. Our cognition is accordingly subject to our a priori [synthesis or] combination (*conjuntio, Verbindung*): objects [of cognition] [themselves] are [mathematically] composed (*compositio, Zusammenhang*) and [dynamically] connected (*nexus, Verknüpfung*) with each other [in experience]. This is precisely the contribution that we make [to the matter of senses] [and that thus makes *the* experience] and that we cognize therein [in objects] a priori.

[38] The learned distinction between *Gegenstand* and *Objekt* is a matter of doctrinal controversy in Kantian scholarship. Many scholars admit to not [being able to] identify a consistent differentiation between these terms [as Kant used them], sustaining that Kant did not use the pair in a manner that warrants or requires a proper differentiation between them. Many scholars, on the other hand, do adhere to a sort of distinction between these, yet not in unison as to how to distinguish them. The present interpretation in this study, as shown in the paragraph (*sup*.) is a weak one, and although the author of this thesis himself otherwise makes the distinction, it is employed here solely to distinguish between the [phenomenal] object (*Objekt*) *qua* concept and that ob–ject (*Gegenstand*) which corresponds to this in intuition [and in doing so, to avoid any confusion in the text]; and also to stress the tension as not to overlook and miss it. This discussion would otherwise exceed the scope of this paper [as for many other considerations that have been treated *passim*] and require an examination altogether on its own. At any rate, for the purposes hereof, the term ob–ject (*Gegenstand*) is used only where it expressly refers to that which corresponds to [object] in intuition. Otherwise, the term object is used throughout the text [in addition to the phenomenal object] also to convey an ordinary and non-technical sense of the word.

3.3.1. Remarks on metaphysical bearings of syntheticity

The most basic expression of [the doctrine of] syntheticity [of judgments] is that not all predicates which [possibly] belong to a concept [of the object] [*qua* subject] are [necessarily] contained therein. Granted, those predicates which belong to the concept [of the object] contingently are extra-essential marks and hence not contained in the concept [of the object]. Incontestably, they are predicated [unto the subject] in synthetic [a posteriori] judgments. Nonetheless, the doctrine of syntheticity stipulates that also those predicates which belong to the concept [of the object] necessarily [hence a priori], are not *all* contained therein. That is to say, not all necessary truths are derived from [mere] concepts as per the principle of contradiction alone [instead they are imposed on [or synthetically predicated to] the concepts as per the a priori principles of understanding];[39] and further, [even] the principle of sufficient ground [*qua* ground of being] cannot be sufficiently grounded in mere concepts [for ground of being is distinct from ground of knowing (*inf.*)]. For otherwise the former principle would have been only of a limited use; whereas the latter principle would be misused.[40] This thesis finds its full manifestation and confirmation in the metaphysical framework of the *Critique of Pure Reason*, *i.e.,* transcendental philosophy (*q.v.* fn. 37).

The first basis thereof is the duality in human cognition, which is of two heterogeneous and positive sources, *i.e.,* sensibility and intelligibility.[41] This dualism leads Kant to distinguish between the ground of knowing (*ratio cognoscendi*) and the ground of being (*ratio essendi*). Accordingly, ground of

[39] In this respect, the doctrine of syntheticity is a direct blow to Leibnizian panlogicism and Wolffian dogmatism which have all necessary truths as analytic.

[40] Although Leibniz treats principle of contradiction and principle of sufficient reason as two separate principles; Wolff, Baumgarten, and Meier derive [albeit not in the same way] the latter from the former, and thereby derive all necessary truths from the principle of contradiction, or what is the same, 'analytically' from mere concepts.

[41] Kantian dichotomy between concepts and intuition marks the first step of its fundamental departure from the preceding rationalist and empiricist epistemological traditions, which likewise regard these units on a continuum but treat [either] one of them as a negative source.

knowing [a proposition] is [logical] and it is different from ground of being [of a thing] which is real, and accordingly must be grounded in sensibility [for the real is that which is given to sensibility].[42]

Still, the mere duality *de mundi sensibilis atque intelligibilis* of 1770, alone, [albeit necessary] is not sufficient to warrant the institution of [a new] distinction between analytic and synthetic judgments [in addition to that between a priori and a posteriori].[43] Indeed, an *absolute* partitioning of sensibility and understanding would have adequately corresponded to two classes of judgments [and not more], with empirical judgments of experience on one side, corresponding to [sensible] objects of experience on real grounds, and a priori concepts of reason on the other, corresponding to [intelligible] objects of reason in themselves, on [purely] logical grounds [in compliance with the early modern models (*q.v.* fn. 11)]. Therefore, were it not for synthetic a priori cognitions,[44] the [new] distinction would be only trivial as analytic and synthetic judgments would have perfectly matched up with a priori and a posteriori judgments, and therefore would not have been essentially different from the typologies of either empiricist or rationalist models, which were in

[42] Originally the term principle (*arche*) signifies equivocally both the ground [of being] (*Grund*) and the reason [of knowing] (*ratio*) and it was taken as such by the dogmatist tradition, eliding this equivocality. The distinction between the logical and real grounds of cognition (1763) [and previously, grounds of knowing and being, or antecedently and consequently determining grounds (1755)] thus was Kant's earliest and main criticism against the tradition. *See* Caygill, 217; *q.v.* §1. "Kant's subsequent philosophical work is an attempt to systematically think through the implications of rejecting the unity of logical and ontological grounds," [*i.e.,* grounds of knowing and being]. (Idem, 218)

[43] This sheds light on why the distinction between synthetic and analytic judgments has not been developed until 1781, despite the fact that Kant distinguished between the grounds of being and the grounds of knowing as early as 1755, and [as more generalized and perfected] logical and real grounds as early as 1763; or that he identified causality as [a] real [relation] and mathematical cognition as both synthetic and a priori; or that already distinguished between the mathematical and the philosophical by 1764; or that have increasingly and commonly used analytic and synthetic to qualify method, definition, and concept formation; and even after 1770 where his philosophy has attained the semi-critical maturity, with anointing space and time as pure intuitions—which constitute [almost] all of the tenets required for introducing synthetic judgments. (*q.v.* §1)

[44] Against the objections of the dogmatic camp, (see, Beck, Ibid.) this alone warrants the distinction legitimate novelty. (*q.v.* §11)

disagreement only where to draw the line but not as to the line itself, as if they were the two sides of a coin.[45]

In order for synthetic a priori cognition to even come to fore, to suggest itself, there must be a *certain* correspondence, a certain *accordance* between these two spheres of cognition, and this marks the second step; thereby also coming into being of critical philosophy.[46] Our cognition is a synthesis of [sensible] concepts and [intelligible] intuitions. A priori concepts of understanding can only likewise refer to intuitions and thereby have objective reality, *i.e.,* synthesized on real grounds a priori. Thus, intelligible universal predicates of being of previous ontology become the categories of knowing of Kantian epistemology.

In this light, our a priori cognition [of objects], which is merely our contribution to and combining sensible matter, cannot thereby arise [*i.e.*, extracted] from mere concepts logically but can only be imparted thereupon on the real grounds of intuition. In other words, [a priori] necessity does not lie merely in the concept [principle of contradiction] but grounded in and imposed by the [a priori principles of] understanding.

Consider, then, the proposition <X is an object.> Now, object is the highest genus, (28 and 29: *passim*) hence *contains* all concepts [both really and logically] including the concept of X. However, the general concept of object [along with *its* categories] must have objective reality. Therefore <X is an object> can only be *sufficiently* grounded in <X is an ob–ject.> Accordingly, the predication require confirmation from intuition, and it cannot be derived from the concept of X by principle of contradiction alone, although the concept of X is contained under it. (*cf.* §3.4) Such confirmation cannot be obtained by the [logical] principle of

[45] This explains why logic even need not know the name of analytic and synthetic judgments. For regardless of whether it is essentially a logical distinction, it does not arise out of logical exigencies [but real [or transcendental]].

[46] *q.v.* Letter to Herz (10:129–35)

sufficient reason, either, but can only be derived from [a priori] principle of [real] ground, which is itself a synthetic principle, which is none other than a synthetic principle of understanding.

3.4. General remarks

Despite a revolution in metaphysics under the title of transcendental logic, (*q.v.* 3.3) Kant largely inherits the framework of [Aristotelian] [general] logic, as cultivated and passed on by [the Leibnizian–Wolffian] tradition; and employs its classical [discursive] *elements* [*i.e.,* forms of thought] for the technical articulation of critical philosophy. Now, to be sure, what logic does not provide as to adequately ground this new epistemology, Kant devises and has transcendental logic provide. Be that as it may, transcendental logic, contrary to what the name suggests, is properly [a part of] metaphysics [and not logic]; (*q.v.* fn. 37) and it does not on this wise override the mechanics of [the elements of] general logic [nor is it intended to do so].

The discord [between the 'classical' logic and the 'new' metaphysics] is materialized in that the epistemological thesis of the *Critique*, *viz.,* synthetic a priori cognitions, is predominantly haunted by essentially logicist reservations. This is all the more interesting, considering that the immediate criticism of the synthetic a priori by the [metaphysical] dogmatists (Eberhard and Maaß et al.) is made on also logical, and not metaphysical, grounds. However much this is [arguably] caused by the Wolffian elision, (*q.v.* fn. 42) or simply those critics' own failure to distinguish between the real (*read* metaphysical) and the logical; (Bxxxvii, A44/61; 8:187–251 *passim*; *see also* §3.3.1) the locus of the criticism remains to be the [logical] doctrine of essence. Accordingly, even if such [strain of] criticism is trivial, unwarranted, or even misguided, the mere fact of their nature can be meaningful [in the context of the inquiry hereof].

One criticism, which is expounded by Maaß, addresses the specific issue of deciding whether a given proposition is analytic or synthetic, arguing thereby against the [vagueness of] formulation rather than the distinction itself [as that which is formulated]. The point is what is contained [or thought] in an [empirical] concept may change from one person to another as well as it may change from one time to another. (*see also* A727/B755 f.) Accordingly, what is synthetic for one or for one time may be just analytic in another, and vice versa, due to the uncertain boundaries of empirical concepts. (*q.v.* 32a) Likewise, appealing to the earlier generic functions of analyticity as subtractive and syntheticity as additive and their reciprocity and convertibility, a [synthetic] concept can be first appended some mark, and then it can likewise be analysed [in a judgment] as if to reverse the concept formation. This simple but legitimate question is countered by Schultz [with Kant's confirmation[47]] with a response equally simple and legitimate: in such a case, two such concepts would have been different concepts.

The other criticism, by Eberhard, presents a case that concerns the attributes of a concept [and its connection to essence]. Now, attributes of a concept are derived from the constitutive marks, and together they constitute the essence of a concept. (*q.v.* §3.2) Now, in consideration of that attributes are derivable or grounded from constitutive marks, they are all thus contained [even covertly] in the concept, and in accordance with Kant's formulation, and they all must be properly predicated in analytic judgments [claims Eberhard]. Kant sees in this criticism a misuse or abuse of principle of sufficient reason, *i.e.,* a failure to distinguish, again, between logical and real grounds. Because, according to Kant, the issue is not whether an attribute is derivable from the constitutive marks but whether such this attribute is derivable [logically] by means of the principle of contradiction—hence analytically. Therefore, such a criticism, then would only serve to displace the real (*eigentlich*) question from whether a judgment is analytic or synthetic to whether an attribute is analytic or synthetic. Consequently, this criticism, as is, is indeed as unfortunate as

[47] *See* Allison and Martin.

Kant complains. However, Kant's response is still important in that it officially bestows primacy on the criterium [of derivability by] principle of contradiction over the [generic] criterium [mere] of conceptual containment (*cf.* §2.4) in the explicit formulation of syntheticity and it therefore can be considered as a minor revision thereof.

Much as this second criticism is unwarranted and unfounded, the logical architecture of conceptual containment (*q.v.* §3.2.1) seems to generally allow a certain strand of misgivings if not this or that particular one. Now, namely at such general level, the highest concept (28:543 ff.; 29:811, 29:960 f.) under which all concepts are ordered is the concept of an object [in general], and immediately thereunder lie *its* predicaments [*i.e.,* categories] and then predicables and so on and so forth. Notwithstanding that these *ancestral concepts* are a priori and thus be contributions of our spontaneity, they are nevertheless precisely those simple [logical] concepts at which any concept should arrive given enough [logical] abstraction. For mathematical categories are constitutive of the object and to that extent they are necessarily contained in the [also logical] concept [of such object], as already composed (*zusammenhängt*) therein, and imposed along with its formation, *viz.,* unity of the synthesis of the manifold. (*q.v.* fn. 48) Then it follows that, analysis of any given concept, *i.e.,* through analytic judgments, would yield these ancestral concepts solely from the principle of contradiction, for indeed what is contained in a concept (intension), it is contained under (extension), that is, those ancestral simple concepts should be contained in any given concept, and essentially, and for this reason their negation would yield a contradiction. *E.g.,* <X is an object> is to follow from the [mere] concept of X [or, if it is dubious that the principle of contradiction is sufficient in this case, then consider <X is something>]. Now, so be the case that we instead derived this from the principle of sufficient reason; and that this was due to our confusion in distinguishing between *ratio essendi* and *ratio cognoscendi*. Granted, it is certainly true that X *being* an object is a real proposition, as underlined strongly by Kant *contra* Eberhard *et al*., and indeed we had just conflated the real ground with the logical ground. Nevertheless, alas, the very

general problem lies precisely here. That is, the logical framework of conceptual containment categorically allows such conflation, and therefore, it must be considered, is inadequate, as is, to support the epistemological model of synthetic a priori cognitions. However, such inadequacy not in the least begets prejudice to the otherwise standing of synthetic a priori as an epistemological thesis but could merely effectuate a criticism of its [explicit] formulation [and not what is thereby formulated], or of the logical framework underlying it.

At any rate, this general discord further gives way to various other considerations as well, which would on the contrary injure the currency of the formulation to the detriment of what is formulated as well. The line of counterargument in continuance of the general problem raises a question as to that in the series of consecutive conceptual containment, how, *e.g.,* the immediate constitutive marks of concept is then unproblematically analytic. That is, how is it that what *would* make <[cat] is something> synthetic, makes also <[cat] is an animal> analytic, if subjected to the very same criterion, in the view of that both predicates belong to the concept of cat as its *constitutive* marks and would follow from its *definition*? For if the former ground is real, is it not same for the latter ground which partakes on the same series as the former?

Regardless, admit and consider instead <X is something> as both logically analytic [as following from principle of sufficient *reason* [of knowing], hence from the principle of contradiction (*cf.* fn. 40)] and really synthetic [as following from the principle of sufficient *ground* [of being] which is a synthetic principle]. The same proposition then would be considered as either analytic or synthetic depending upon the determining ground that would qualify the proposition. Such an implication refers a potentially problematic class of judgments, *i.e.,* whereby mathematical categories are predicated on the subject, which would resist an absolute analytic–synthetic distinction as presently formulated within the logical framework and

which would have to be admitted into both types.[48] At that, this deliberation would thereby contribute to the controversy as to whether the distinction is logical or epistemological, however, regrettably, not to its resolution.

[48] It is clear that dynamical categories should present no complication, for one, real relations do not inhabit the object, and for they already would be predicated to concepts [of the objects], synthetically and as given in experience. So that our judgments with respect to them would be synthetic a posteriori without any reservation.* However, mathematical categories entail an asymmetric bifurcation in the conceptual architecture for they are both logically necessary [that is mathematical marks are necessarily contained logically in the concept] but they also have objective reality. In other words, their logical and real necessity overlap. Accordingly, with respect to mathematical properties of the concept, there is no marker as to whether [any] judgment is referring to logical or real plane. That is, *e.g.,* one has to specify whether one is speaking in logical terms or real terms.

* Although experience is combined by us a priori, what we establish in experience would be a posteriori. *E.g.,* that *X* causes *F* is a synthetic a posteriori judgment, although the concept of cause is an a priori concept. In other words, it is not necessary [or a priori] that [this or that] *X* causes [this or that] *F* but only that I think [cognize] this relation as a *causal* relation. That is to mean, that I necessarily have the concept of cause [which only objective application]. Hence the natural sciences being a posteriori (a priori *secundum quid*), and only their fundamental propositions (*physica pura*) [which are the synthetic principles of the understanding] being a priori.

4. MATHEMATICAL FRAMEWORK[49]

Avowedly, mathematical propositions embody the paradigm for synthetic a priori judgments, and to that extent is of direct pertinence to [the demonstration of] their possibility. Moreover, mathematics also plays a further key role as a model [science] [for metaphysics] in extending [hence synthetically] cognitions [of reason in speculative or transcendental use] [securely and entirely] a priori. (Bx ff., A4/B8 f., A712/B740 f.)

Although Kant did not develop a stand-alone theory for mathematics, the idiosyncrasies of and the relation between mathematical and philosophical cognitions were ever-present in his thought; and these inquiries are gradually developed into one of the constitutive themes of the first *Critique*.[50] This is articulated in the question "how is pure mathematics possible" (B20) as one respect of the general problem.

4.1. Mathematics in general

In Kantian terms, mathematics exhibits the characteristics of both doctrine (*Lehre*) (complex of rules) and science (*Wissenschaft*) (complex of cognitions). On one hand, it is a doctrine, for, as a demonstrated discipline, it provides the principle of

[49] This particular framework for mathematics take place *passim* in the first *Critique*: otherwise general or occasional remarks aside, *Transcendental Aesthetic* gives the source of [pure] mathematics; *System of All Principles* grounds the possibility of [pure] mathematics and its application to nature and experience; the most substantial and direct treatment is found in the *Discipline of Pure Reason in Dogmatic Use* which progresses in a fugal fashion in definitively juxtaposing mathematics and philosophy; finally, the *Schematism* complements the framework for mathematics, also completing the full epistemological model of synthetic a priori.

N.B. Schematism and *System of All Principles* treat respectively the pure concepts and the [synthetic] principles of [pure] understanding. Their [general] provisions, to that extent, are applicable to [applied] mathematics [and then] only relatively. *See also* fns. 52, 58.

[50] Along the lines of his *criticism* of the *dogmatic* metaphysics [and its claims to *mathematic* certainty], Kant juxtaposed the mathematical and the philosophical as early as in *Negative Magnitudes* (1763) and the *Inquiry* (1764) and developed the difference into a main tenet of *critical* philosophy as well as the idea of synthetic a priori cognitions. (*q.v.* §1)

production [of cognition] and rules which must precede such product. On the other hand, it is a science [of reason], for it is the complex of cognitions [from principles] as a system [as opposed to common cognition, which is the complex of cognitions as mere aggregate]. As a doctrine, mathematics [must be[51]] the *canon* of the [pure] sensibility [as to form]. As a science that contains the ground for the extension of our cognition in regard to mathematical use of reason, it is also an *organon* [through which mathematical cognition becomes possible as to content.] (9:12 f., 9:72; 24:694f., 24:797; A796/B824)

Mathematics, as a science, has two uses recognized by critical philosophy: first, as pure mathematics, it is an autonomous science of pure reason that is alone able to produce entirely pure cognitions;[52] second, as applied to [possible] experience, it enables the sound practice of and lends [a priori] security to natural sciences. Kant also identifies under mathematics further the branches of algebra, arithmetic, and geometry; characterizations of which depend on their mode of relating (*Beziehung*) to the mathematical objects.

4.2. Mathematical cognition

Mathematical cognition refers to a specific mode of cognition (*modus cognoscendi*) of reason [that proceeds] from the construction of concepts. To construct a concept is to exhibit *a priori* [in] the intuition [the ob–ject] that corresponds to [that

[51] *N.B.* This is not advanced by Kant, but it follows from his theory of science as explicated in logic and metaphysic lectures. (Ibid.) This point will be further developed. (*q.v.* §4.4)

[52] Although [pure] mathematics is a cognition entirely pure and a priori, and it cognizes securely so without any licence of objective validity, its possibility as such must nevertheless be deduced and its evidence be made comprehensible in transcendental philosophy. (A733/B761) For, it would have been nothing but an "occupation with a mere figment of the brain," if space were not the condition of appearances. Therefore, pure synthetic judgments [of mathematics] are still related, albeit only mediately, to at least the [possibility] of experience. (A157/B196) Thus the possibility of [pure] mathematics as well as its secure application to appearances are established in the *System of Principles*.

concept].[53] The construction of a concept therefore requires a pure intuition,[54] which in turn, as an intuition, is a particularly determined, individual object. But, as the construction of a concept, this intuition also conveys universal validity for all possible intuitions that fall under the same concept, insofar as they fit the schema, and to that extent regardless of any contingent determinations. (9:23; A713/B741 f.) To illustrate:

> I construct a triangle by exhibiting an ob–ject corresponding to this concept [... in intuition]. [The triangle] [...] serves to express the concept without damage to its universality for in the case of this [...] intuition we have taken account only of the action of constructing the concept, to which many determinations, *e.g.,* those of the magnitude of the sides and the angles, are entirely indifferent,[55] and thus we have abstracted from these differences, which do not alter the concept of the triangle. (Ibid.)

In this regard, unlike the philosophical [mode of] cognition which considers the particular in the universal, the mathematical [mode of] cognition considers the universal in the particular, for those universal conditions of construction which determine the individual [singular intuition] determine also universally the ob–ject [[of the concept] to which it corresponds]. (A714/B742)

Mathematics is the science of magnitudes and the mathematical [mode of] cognition likewise pertains only to magnitude, [the concept of] which alone can be

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[53] There are various forms of this philosopheme *passim* the Kantian corpus, variety of which is likely to cause confusion as to what exactly is constructed or exhibited, and so on: *e.g.*, constructing the intuition (*e.g.,* A716/B744), concept is exhibited (*e.g.,* A721/B749), magnitude is constructed (*e.g.,* A717/B745), object is constructed (*e.g.*, Ibid.) intuition is constructed (*e.g.,* A716/B744), exhibited in intuition (*e.g.,* A714/B742), *etc.*

This elasticity is confusing when it is due to equivocality of the terms but illuminating when it arises out of contextual synonymity and terms are duly intertranslated (q.v. dynamical analyticity). To revise the formulation, it is the concept what is constructed. The ob–ject that corresponds to this concept [in intuition] is [of the kind of] a magnitude. The magnitude *qua* ob–ject (*n.b.*, passive case) is exhibited [*i.e.*, we have it exhibited] in [the course of] *intuit-ing*. Inasmuch as [magnitude *qua* ob–ject] is intuited, it also thereby constitutes an *intuit-ion* [as that which is exhibited]. Also, when we exhibit the ob–ject, we also thereby have the ob–ject made, *i.e.* constructed. Finally, when the concept (*n.b.*, dative case) is [being] exhibited [to], the concept is thereby [being] provided with the so–exhibited [that is, made, or constructed] ob–ject [or, what is the same, intuition *qua* ob–ject].

[54] For from empirical intuitions, [empirical] concepts are *described* but not *constructed*. (*q.v.* §3.3)

[55] Mathematical construction is thus a production of [relative] proportions of magnitudes [and not absolute units]. *See also* §4.4.

constructed, *i.e.,* exhibited *a priori* in intuition. Mathematics construct magnitudes (*quanta*) [in geometry] and mere magnitude (*quantitatem*) [in algebra [and for arithmetic]]. In the former, *viz.,* ostensive construction, one constructs the ob–ject, as such, that corresponds to the concept [of this ob–ject]. In the latter, *viz.,* symbolic construction, one entirely abstracts from the constitution (*Beschaffenheit*) of such ob–ject, and through a certain notation (*Bezeichnung*) [which is employed for the construction of magnitudes in general [*i.e.,* numbers] and for their potential relations (*Verhältnis*) [*i.e.,* in mathematical operations for generating and altering [these magnitudes]] designates (*beziechnet*) the general concepts of these magnitudes in accordance therewith. It thus exhibits in intuition all the procedures through which the magnitude is [to be] generated [and altered] in accordance with certain rules [*i.e.,* of mathematical operations] by means of their symbols.[56] (A717/B745; R5583)

Still, in order for a determinate magnitude to be recognized by the understanding in its determinate concept as that particular determinate ob–ject, a schema is required for the manifold [of magnitude] to belong, and hence belong to its concept. Indeed, the individual [singular intuition] which is determined [under certain universal conditions of construction] corresponds to the ob–ject of the concept [to which this individual corresponds] only as its schema, which provides the essential manifoldness and order of the parts that is determined a priori from the principle. (A714/B742, A718/B747, A833/B861)

The schema is a product of the imagination but since the synthesis of imagination aims at no individual intuition [but rather only the unity in the determination of sensibility], the schema is to be distinguished from an image. Schemata provide concepts with their images. (A140/B179) In this regard, it is not images of objects

[56] On this account, it can be argued that Kant, analogous to Greek mathematical tradition, regards geometry as occupying a prime position relative to algebra and arithmetic, for these deal symbolically with, or, they are symbolic of, the magnitudes [of ob–jects themselves] which are ostensibly constructed in geometry.

but schemata is that which ground our pure sensible concepts.[57] The schema for sensible [*a priori*] concepts, *i.e.* mathematical concepts, is a product of pure *a priori* imagination, through and in accordance with which, first, the image becomes possible; then, the image is connected with the concept, to which they are in themselves never fully congruent, by means of the schema that [image] designates.[58] (A140–2/B179–81)

Pure image of all magnitudes (*quanta*) for outer sense is space. Pure schema of quantity (*quantitas*) is number which is a representation of the successive addition of one homogeneous unit to another in time. Pure schema of quality (*quantum*) is the magnitude insofar as it fills time, that is, the continuous and uniform generation of that magnitude in time. Through the former we conceive discrete magnitudes, through the latter, continuous magnitudes.[59] (A142/B182 f.)

4.3. The mathematical v. the philosophical

Our cognitions can be distinguished either [objectively] with respect to where they arise from, as rational and empirical cognitions; or [subjectively] with respect to how they are acquired (*erworben*), as rational and historical cognitions. In the objective respect, every cognition that comes through experience is empirical and

[57] "If I place five points in a row, this is an image of the number five. [...] if I only think a number in general, this thinking is more the representation of a method for representing a multitude in accordance with a certain concept than the image itself [...] No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept which makes this valid for all triangles, right or acute etc. but would always be limited to one part of this sphere [...] The schema of the triangle can never exist anywhere except in thought and signifies a rule of the synthesis of the imagination with regard to pure shapes in space." (A140/B179 f.)

[58] It must be kept in view that *Schematism* is concerned with the pure concepts of understanding and transcendental schemata that would function as a medium to ensure homogeneity for the application of the categories to appearances, sensible intuitions of which the former is entirely heterogenous from. However, other concepts [—mathematical included] are not entirely heterogonous from the ob–jects they conceive, and for this reason does not require such transcendental schemata. (A137/B176 f.) In this regard, provisions of this section hold only relatively and in principle for mathematical cognitions.

[59] Although numbers are generated in and through time, time is not to arithmetic as what space is to geometry, for the number is a unit of magnitude, and not the magnitude, and arithmetic is thereby not exclusively the science of time. *Cf.* fn. 56.

that originates from [pure] reason is a priori. In the subjective respect, rational cognitions are from principles (*ex principiis*), *i.e.,* drawn from grounds a priori, whereas historical cognitions are from data (*ex datis*), *i.e.,* acquired [mechanically] a posteriori.[60] Mathematics and philosophy [are only substantial sciences[61] that] are likewise cognitions of reason in both two respects, albeit still radically different. (9:22 f.; 24:693 f., 24:797)

Mathematics and philosophy differ not in their [so-called exclusive] objects [which is only the effect], *i.e.,* quantity and quality, but in their procedure, *i.e.,* use of reason [which is the cause of difference].[62] For indeed philosophy treats everything, including quantity; likewise, mathematics deal with everything insofar as it *has* a quantity. Although they may have common objects, their procedures of cognizing these are entirely different. While philosophy proceeds a priori exclusively through concepts; mathematics proceeds through intuitions which it exhibits a priori for concepts. Accordingly, in consideration of their respective determining grounds of cognition, (*q.v.* fn. 23) mathematical cognitions are intuitive while philosophical cognitions are *merely* discursive. (A714/B742 ff.; 9:22 f.)

Thus, the difference between the mathematical and the philosophical is essentially that of two distinct [rational] uses of reason, as intuitive and discursive. This twofold use of reason pertains to the constitution (*Beschaffenheit*) of appearances through which ob–jects are given to us: form and matter. The form, *qua* conditions of space and time, is contributed by intuition entirely a priori, while the matter belongs to sensation and determined only a posteriori. Hence, as to matter, we cannot have any

[60] Subjectively rational cognitions are only those that originate [objectively] a priori. However a priori, *i.e.,* objectively rational, cognitions can be acquired (*erworben*) either rationally or historically, which in the latter case they are to be of merely 'imitative' or 'mechanical' use. Empirical cognitions, as they are originated from data, can only be acquired historically. (9:22)

[61] Substantial cognitions are those that properly of objects. Logic, on the other hand, while rational on both accounts, is not substantial [but merely formal].

[62] It is therefore the way they relate to their objects is what dictates which objects they relate to, that is, while quantities can be constructed a priori in intuition, qualities must be exhibited in empirical intuition. (Ibid.)

a priori concepts except for the indeterminate concepts to synthesise [possible] sensations. Such is the discursive use of reason; it can do nothing further than to bring appearances under those [indeterminate] concepts according to their real content [which cannot be determined except a posteriori]. However, as to form, we can determine concepts a priori in intuition, by composing the ob–jects *qua* magnitude in space and time through homogenous synthesis. This, then, is the intuitive use of reason, because these concepts can be determinately given in pure intuition a priori. (*Recited from* A719–24/B747–51)

In light of this, to determine whether or not something exists and for how long [*i.e.,* quantity and quality], or whether it is a substratum or a mere determination, or how it is related to other things [*i.e.,* relation], or how it relates to its existence [*i.e.,* modality] is a rational cognition from concepts, and is called philosophical. Whereas, to determine an intuition a priori in space and in time or its magnitude is a concern of reason through construction of the concepts and is called mathematical. (*Recited from* A724/B751)

Discursive elements: definitions, principles, demonstrations

From the side of understanding, mathematical cognitions, like any other cognition, proceed through the discursive elements, all of which it rationally utilizes, however in a manner different from philosophical cognitions. (A726/B754)

Mathematical concepts are [originally] made and can be *defined*, whereas philosophical concepts are given and cannot be properly *defined* but only *exposed* [and thereby explained]. Mathematical definitions come about through synthesis and is therefore complete, philosophical expositions come about through analysis and is never [certainly] complete. (A728/B756 ff.; q.v. fn. 32, §3.2)

Mathematical definitions can never err, for the concept is given through the definition and contains only that which the definition gives. Analytic definitions,

however, can err either by including marks that really do not belong or by lacking exhaustiveness. Accordingly, mathematical method can and must proceed from definitions, for indeed it would otherwise have no concept to begin with; whereas philosophical method cannot put the definitions first for it has none, rather it seeks to attain them at the end by completing the exposition. (A731/B759 f.)

A priori principles can be either intuitive or discursive. The former can be exhibited in intuition and are *axiomata*; the latter is advanced only through concepts and are *acroamata*. (A148/B188; 9:110) *Axiomata* are immediately certain [synthetic] *a priori* principles. Since mathematical principles are derived from intuition and [never from [general] concepts], it is only mathematics that is capable of axioms among [rational] cognitions. *Acroamata* are just as certain, nevertheless only mediately, since they require a [third] medium [to subordinate [propositions] of [possible] experience]. Accordingly, while *axiomata* are self-evident, *acroamata* require deduction, through which philosophy must justify their authority.[63] (A25/B41, A137/B176 ff., A732/B760 ff.)

While principles are immediately certain, thus indemonstrable; demonstrable propositions that require and are capable of proof are called theorems. Essential moments of every theorem are thesis and the demonstration. Thesis is the proposition that is drawn through a series of consequences from principles. Only a proof that is made fully a priori, *i.e.,* an apodictic proof, insofar as it is also self-evident is called a demonstration; and only and apodictic proof that is intuitive can be self-evident. From a priori given concepts self-evidence can never arise, regardless of its otherwise apodictic certainty. Therefore, it is only mathematics that contains demonstrations, for it derives its certainty from the a priori intuition. Here the relation of magnitudes is displayed in intuition, whether symbolically or ostensibly, and all inferences are secured against mistakes by fully exposing each

[63] N.B. In the *Critique of Pure Reason*, 'pure synthetic [a priori] principles [of sensibility]' refer to mathematical principles and 'synthetic [a priori] principles of pure understanding' refer to philosophical principles.

inference. Mathematics thereby examines the universal in concreto in individual intuition, where every mistake becomes immediately visible. Philosophical cognition, alas, forego such a display, for it can only consider the universal in abstracto through the concepts. and yet through pure a priori intuition. Philosophical proofs, on this ground, are called acroamatic proofs as to indicate that they cannot be conducted except through 'words' alone. (9:110–2; 24:694; A734/B762 f.)

All apodictic propositions are *dogmata* as they belong to philosophy or *mathemata* as they belong to mathematics. An immediately certain synthetic proposition through concepts is a dogma, while that obtained through construction of concept is mathema. (24:831; A736/B764) Pure reason in its speculative use contains no immediately certain synthetic judgment from concepts. For ideas are not capable of any objective validity; and, pure concepts of the understanding, while they are certain, they are only mediately through the relation of these concept to [possible] experience which is contingent. Therefore, synthetic principles from these concepts cannot be a dogma. Now, if the speculative use of pure reason contains no dogmata, then any dogmatic method, too, would be inherently inappropriate. (A736/B764 f.; 24:831)

CONCLUSION

Kant is not particularly concerned with an ontology of mathematical entities. Now, to be sure, these are objects of our reason [or, to be exact, as pure intuitions of pure magnitudes, they are objects of our [pure] sensibility as a faculty of reason]. At that, it should be also underlined that, by exhibiting the corresponding ob–ject in intuition, mathematical cognition constructs not only concepts [of objects] but [their] ob–jects as well. This is a notable exception, for among our [speculative] cognitions only in [pure] mathematics ob–jects are not *given* [to us] but *made*.[64] In this regard, it is evident that mathematical entities are not things in themselves whose *reality* we are equipped not to ever cognize. They are not appearances [of such things in themselves] either. Any appropriate designation aside, these entities [purely intuit–ed magnitudes] can be considered tentatively, for the purposes hereof, as *sui generis* things [for us]. Their unique ontology suggests, at least, within the critical context, these things should be exempt from the absolute inaccessibility; so should be our reason from any cognitive limitations [with respect thereto].

The principal critical doctrine is that reason cognizes [a priori] only itself [*i.e.*, its own design]: either it cognizes [in things] what it itself puts into them in the way it so puts (*i.e.,* combination: composition and connection), or else it cognizes what it itself makes and how (construction). In the former case, it so cognizes [mediately by the objective application of pure concepts], that is, inasmuch as it is spontaneous. In the latter case, it [should be able to] so cognize [immediately] all the properties [thus be entirely spontaneous]. Indeed, reason has [full] insight into, *i.e.,* comprehension of, only what it makes, and whose truth therefore he is certain

[64a] Not to be confused with those fictitious objects which correspond to no ob–ject in [either pure or empirical] intuition hence yield no cognition.

[64b] *Cf*. Bx "Mathematics and physics are the two theoretical cognitions of reason that are supposed to *determine* their objects a priori, the former, entirely purely … " and Bxi "[rational] cognition can relate to its object in either of two ways, either merely determining the object and its concept (which must be given from elsewhere), or else also making the object actual. The former is theoretical, the latter practical cognition of reason."

throughout. (*q.v.* Diagram 2) For indeed, "nothing can be comprehended more than what the mathematician demonstrates." (9:65) Since these are not given to us but made, and we [can] only comprehend (*begreifen*, *comprehendere*) what we ourselves make, these ob–jects are thereby both thoroughly determinable and thoroughly determinate[d]. (*q.v.* 58b)

Accordingly, the construction of ob–jects draws attention to that while our intuit–ing is in principle passive, it is active in [pure] mathematical cognition. That is to mean, reason is spontaneous in that it produces its own [intuitive] representations [without representations being effectuated by some–*thing* else]. Does this suggest an exclusive intellectual intuition in the realm of mathematical cognition [which is space and time], or an intuitive cognition—one that does not merely designate the determining ground of cognition? (*cf.* fn. 23) Nevertheless, this is certainly not admissible, for intellectual intuition would not require the concept [at all] as a reference and would not distinguish between a sense (*Sinn*) and a reference (*Bedeutung*) either.

Still, [all] our cognition is [ultimately] discursive, because other than by virtue of the concepts, we are not able to cognize the indefinitely determined individual as [a limited, thus determined] object [of cognition]. The concepts, as rule functions, are therefore necessary instruments which allow us to pick marks up out of an indefinite multitude [of marks of things];[65] thereby provide our cognition with both sense and reference. In light of this, since Kant's account of human cognition is not psychological, the hypothetical question is warranted: what is the use of concepts in mathematics, if mathematical objects were [at the same time for us and] in themselves and were already thoroughly determined [or determinable] by us, as is?

Such a query would be twice relevant in the context of this study, for it is the logical architecture of our cognizing, *i.e.* [logical] form of concepts and judgments, that

[65] " … while the intuition provides a field within which the manifold of intuition may appear as a representation, it is the concept which synthesizes these representations into experience and knowledge." (A68/B93).

underlies the formulation of syntheticity of judgments (*q.v.* §3.2). For indeed, even if there were no recourse to logical primacy of judgments, in this case, all our cognition [*i.e.,* [sensible] concepts] would still have already been *synthesized* (*q.v.* §§33.3). In short, bottom of the question here would be that whether syntheticity of mathematical propositions are due to the *sui generis* nature of mathematical cognition of reason or else merely to the general framework? Expressed provocatively, are mathematical propositions synthetic because they are mathematical [that is, they proceed through the construction [of concepts]] or because they are propositions [that is, they relate concepts to each other, *i.e.,* a predicate to a subject that does not contain it]?

The tension is perhaps better illustrated by not what we [are able to] do in respect of mathematical cognition but by what we cannot [at least easily] do, *i.e.,* those mathematical propositions which are not readily clear and evident, or as so *demonstrated*, for us. Is it because we fail to cognize, that is, reason does not cognize what is [presented or] given before it [by itself]; or we fail to [duly] make [any such magnitude in the first place]? Or else does this further point to a distinction between [mathematical] cognition (*Erkenntnis*) and knowledge (*Wissen*) parallel to that between a cognition and demonstration, wherein we may cognize and not still know? At any rate, the only possible argument that could be drawn from this idle chatter, the only possible merit, would be that, in this light, there is definitely [a distinction between] a making [*i.e.,* construction] and a cognizing [either of which may *individually* fail in mathematical cognition].

The tension can also be expressed in terms of and as between concepts and intuitions, and it would give us the use of concepts in mathematics (*sup.*). Indeed, concepts are needed in mathematics, for without concepts, what would be constructed in intuition would remain a mere manifold, and not a mathematical object, *per se*. Therefore, what anoints a certain manifold of mere magnitude [as] a certain [*e.g.,* geometrical] figure, or what is the same, what [re]cognizes in this certain manifold such certain figure—and not some other—is the concept of that

figure through the [mathematical] schema. Therefore, mathematics, and at that, [mathematical] categories of understanding as well, require concepts for otherwise the constructed magnitude, *i.e.*, exhibited ob–ject in intuition would not become an object in the first place. Through concepts, therefore, mathematical objects can be referred to (*bedeutet*) and meaningful (*sinnvoll*).

In any case, we do—and this entails and illustrates the discursivity of mathematics alone—conceptualize the same magnitude exhibited in the intuition as different objects. *E.g.,* the same pure intuition may [thereby] correspond to quadrangle, rectangle, or square [which respectively are different concepts].

It is also the case that, further, such concept of square [*qua* species] must contain in it more than the concept of rectangle [*qua* genus], and so on and so forth. Now, [analyticity or] syntheticity of propositions in which the concept of square is subject would call for further consideration on a number of axes:

First, the concept of square is the lowest species (*conceptus omnimode determinatus*) unlike the concept of rectangle or quadrangle. While the genus rectangle or the genus quadrangle are concepts which are common in that they contain under themselves *various* [forms of] magnitudes *qua* shapes. The completely determinate concept of square, on the other hand, while still common [and hence a concept] in applying to only *one* square [for there is only one form of square], have indefinitely many *uniform* instances of the same [form of] magnitude *qua* shape. This peculiarity is due to that in [pure] geometry there is only [relative] proportions, (*q.v.* 55) and not [absolute] sizes, since "space is [represented] as an infinite given magnitude." (B39) Therefore, in mathematical cognition, one should first note these two [distinguishable] types of [mathematical] concepts—a distinction that is not available to the concepts of philosophical or common cognition, which may only admit singular [referential] *use* of a concept.

Second, since we make the mathematical ob–ject and have a priori insight into it, *i.e.,* since it is *sui generis* in itself (*sup*.) and cognizable fully as such, its real essence and logical essence either overlaps [*e.g.,* the concept of square exhausts and perfectly overlaps with the corresponding intuition] or the former already *contains* the latter.

Now, in this light, syntheticity of a certain mathematical proposition should be reviewed. To borrow from Kant his example:

> Thus, the common explanation of the circle, that it is *a curved line every point of which is the same distance from a single one (the center-point)*, contains the error of unnecessarily introducing the determination *curved*. For it must be a particular theorem, which can be deduced from the definition and easily proved, that *every line each point of which is equally distant from a single one* is curved. (A731–2/B759–60; emphases added)

Now, the constitutive marks of the concept circle consist of a <line> <every point of which is the same distance from a single one> whereas being <curved> is an attribute, for it follows from the constitutive marks and thereby belongs to the *essentialia* of the concept of circle. Would the proposition <every line each point of which is equally distant from a single one is curved> be a synthetic proposition; or, more clearly, would <circle is curved> be a synthetic proposition? Although this seems absurd, it is not. Because indeed, one needs to process the <every line each point of which is equally distant from a single one>, *i.e.,* construct it in intuition, and synthetically and *gradually* attain the <curved> line. Nonetheless, the conundrum here, which has brought this case forward in the first place, is different, namely that, when one posits the <circle,> does one posit the definition or the magnitude [first], thereby having already acquired the shape, *i.e.,* the curved line. To illustrate, when I judge a <circle> as <curved,> consider, I happen to have *already* [intuited and] cognized the curve through construction, which would make my proposition analytic [based on the already exhibited magnitude] in an epistemic sense, while it would be synthetic [based on the definition of concept] in a logical sense. The former suggests, insofar as the [mathematical] *concept* [which is fully determinable] is determined, to that extent analytic cognitions may arise.

Perhaps to better shed light on the issue and to discover or invoke other respects, as the case may be, and generalize the possible problematic, if any, another example might be given. Consider the special 3:4:5 right triangle. Hence the definition would be a <right triangle> with a <leg of 3 units> and a <leg of 4 units.> Bear also in mind that I already have what is the shape of 3:4:5 right triangle with its shape and proportions [ostensibly] constructed. Now let the proposition be <a right triangle with a leg of 3 units and a leg of 4 units> have a <hypotenuse of 5 units>. Now, if we compare this proposition with the one above, it is clear that, in respect of ostensive construction I have the magnitude of hypotenuse already constructed, so on this account it is plausible that this proposition would be analytic. However, in respect of symbolic construction, it is *not as implausible* that the proposition would be synthetic, and that I would need to have recourse to intuition in order to attain <5 units>.

Therefore, the second case brings another issue for consideration on top of the former. Should a distinction between be made in respect of analytic–synthetic distinction, *e.g.*, ostensive and symbolic construction [*i.e.*, between geometry and algebra]?

At that, the proportional determination of determinate geometrical objects reveals how algebra *qua* symbolic magnitude relates to and can be grounded in geometry *qua* ostensive magnitude, since proportions *can* be expressed [or symbolized] in algebraic and arithmetic terms.

At any rate, the case of arithmetical propositions, considered alone, requires closer inspection. As the proposition <7+5=12> being the paradigm example for the syntheticity of mathematical propositions (B15 f.), Kant also mentions elsewhere:

> [...] that equals added to or subtracted from equals give an equal are analytic propositions, since I am immediately conscious of the identity of one generation of a magnitude with the other [...] (A164/B204-5)

If we work back the famous example accordingly, it comes to that <12-12=0> is an analytic proposition. This may seem at first too trivial a point, but it invokes further, less trivial, examples that would appeal to the same ground [*i.e.,* 'identity of magnitudes']. Namely, since the magnitude of the number <12> has already been synthesised as <1+1+1+...+1> in counting, what would be the status of the proposition <12=1+1+1+...+1>? Or, for that matter, since I must already have had [constructed] <5> and <7> when I posit <12>, what would this make of the proposition <12=5+7>?

Such examples could be extended to infinity, where the proposition is a simple addition, the sum is the subject, and the addenda [consisting of homogenous units of <1>] assume the function of predicate [as well as variations with other operations, and so on]. However, these would all still be trivial counterpoints, even if they can be plausibly argued to be analytic propositions, in consideration of Kant's *idea* of the syntheticity of mathematical propositions. So, it would prove a worthier effort to look instead for a more general level of discussion, if any, under which all these specific concrete examples could be brought.

Second, that <12-12=0> is an analytic proposition indicates that [general] logic, abstracting "from all contents of [mathematical] cognition and of the difference of its objects" (A54/B78) recognized if not the number *qua* mathematical object, for they were identical regardless, but the [mathematical] operation of subtraction and the function of [mathematical] equality. In any case, the question is does logic recognize the rules of the mathematics, and, if so, what other mathematical operations can it achieve? Alternatively put, would it be that mathematics is a canon [of the pure sensibility] on par not with metaphysics but logic?

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