

CREDIT RISK MODELING, SIMULATION AND CREDITMETRICS
IMPLEMENTATION

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Credit Risk Modeling, Simulation and CreditMetrics Implementation

Kredi Riski Modelleme, Simulasyon ve CreditMetrics Uygulaması

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- 3) Economic Capital
- 4) VaR
- 5) CreditMetrics

ABSTRACT

Credit risk portfolio management is getting more and more attention every day, it was not as common as it is now. Financial crisis, risk manager's expectations of more efficient in credit portfolio management and the regulator's requirements from financial institutions is about the capital management that causes the evolution of credit risk management.

In this thesis, credit portfolio modeling essential parameters for quantifying portfolio credit risk, default risk, concentration risk, and features of credit risk models are analyzed. The Monte Carlo simulation method is employed for analyzing CreditMetrics modeling results. Hypothetical portfolio's expected loss, unexpected loss is also referred to as value at risk, expected loss and unexpected loss of portfolio which has equal exposure distributed loans, credit expected and unexpected loss are demonstrated.

ÖZET

Kredi riski portföy yönetimi her geçen gün daha fazla dikkat çekiyor, daha önce bugünkü kadar yaygın değildi. Finansal krizler, risk yöneticilerinin daha etkin kredi portföyü yönetme beklentisi, regülatörlerin finansal kurumlardan sermaye yönetimi gereksinimleri kredi riski yönetiminin gelişmesinin nedenlerindendir.

Bu tezde, kredi riski portföy modeli temelleri, kredi riski portföyü ölçüm parametreleri, temerrüt etme riski, konsantrasyon riski, kredi riski modelleri özellikleri analiz edildi. CreditMetrics model sonuçları için Monte Carlo simulasyon modeli kullanıldı. Varsayımsal portföyün beklenen kaybı, beklenmeyen kaybı, eşit kredi dağıtılmış portföyün beklenen ve beklenmeyen kaybı, kredilerin beklenen kaybı ve beklenmeyen kayba katkısı gösterildi.

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List of Abbreviations

A-IRB – Advanced Internal Ratings Based Approach

EAD – Exposure of Default

DM – Default Mode

IRB – Internal Ratings Based

LGD – Loss Given Default

MTM – Mark to Market

PD – Probability of Default

PDF – Probability Density Function

VAR – Value at Risk

1 Introduction

When BIS was in place in 1998 with internal models for market risk, the next step was to build a VAR framework for credit risk. The Regulator needs to be confident for “default risk”, “concentration risk”, “spread risk” and “downgrade risk” of credits that they are assured by the institution.

Credit risk includes more issues in contrast to market risk because the typical credit return distributions mostly do not fit the normal distribution. Spread risk is related to both market risk and credit. Spread could fluctuate due to the market conditions which result in the credit spread fluctuations. Downgrade credit risk is a reflection of the spread risk.

Default risk is a special case of downgrade risk. When the credit deteriorates, the obligor cannot pay the amount that has to.

The profitability of companies reflects the condition of market which is affected by exchange rate, interest rates, stock market indexes, unemployment rate etc. Thereby all these changes fully affect the exposure of customer ability to pay, probability of defaults, credit downgrade or upgrades etc. (Crouchy, Galai & Mark, 2000, pp: 59-62)

1.1 Economic Principles of Risk Management

Risk Management abducted an important management decision tool for financial institutions. Recently, this situation differentiated risk management as an eye-catching for risk managers. Appropriate risk appetite based on evaluations as a result of quantitative models of measuring and pricing risk.

A quantitative model for risk management is not a traditional way as usual which covers extreme losses in the new concept. Therefore, Understanding the concept and principle of risk management will help financial institutions to manage their risk especially of large losses and adjust the vulnerability of the firms. The amount of capital behind the institution’s investments and portfolio positions are components of these vulnerabilities. On the other hand, firm’s risk management system, liquidity of positions and many other factors could cause vulnerabilities.

Mostly, reduction of risk is one of the means to decrease the volatility of profit and loss or market value and quantifying the risk depends on the condition of the economy which refers to the demonstration of the investment environment. This environment gives ideas on macroeconomic, political, financial information and so on. Over times, the credit quality of counterparties, price of underlying market indices, volatility of prices and portfolio changes are the needs for risk management.

1.2 Principles of Credit Risk

Banks have not enough information about the borrower's credit risk. Borrowers know more than its lender. In order to adjust the borrower's credit risk and control the disadvantages, banks set a limit for borrowers to access their loan rather than without restrictions. These limits were based on firms' credit quality along with available information about borrowers. For instance high rated (AAA) company could have taken the higher limit than the company which had a lower rate (BBB).

Credit risk limit implementations are not only for individuals but also for sectoral groups, borrowers which are in the same regions and some other classifications.

The fact presently is that credit risk measurement is more important now than it earlier was, thus, its management evolved because of:

- ❖ Increasing bankruptcy frequency,
- ❖ More competitive margins on loans,
- ❖ Reduction in value of real assets in multiple markets,
- ❖ Impressive growth off-balance sheet instruments with inherent default risk exposure including default risk derivatives.

In order to arrive at subjective analysis on how suitable borrowers are to be granted credit or not, some key parameters are taken into account for most financial institutions. Particularly:

- ❖ Borrower reputation is good enough or which level to rely,
- ❖ Borrower capital level or leverage,
- ❖ Firm's earnings or cash flow to pay debt obligation.

Risk managers focus on more complex credit scoring systems using this information as well as relying on them to examine not only individual loans but also analyze the portfolio risk. (Duffie & Singleton, 2003, pp: 12-29)

Risk Managers prefer to use or develop objective models instead of the subjective ones that were used before. Comprehensive and objective model's benefits of risk management cannot be ignored as the reasons are well articulated in this thesis.

Credit Portfolio includes; default Risk, concentration risk, correlation risk, recovery risk, and credit rating migration risk. Each risk is important to be observed in building efficient portfolios but risk managers tend to easily see the default risk and concentration risk effect on portfolio as more crucial than others.

In Credit Metrics portfolio modeling, losses result from rating migration and default event. Concentration risk, correlation and recovery risk contributes on the losses when default event occurs on obligors. They also help to ascertain if the measurement of loss is high or manageable level in portfolio.

The loss can be referred to as an expected loss that the bank expects to lose. Expected loss is manageable in the bank because they have a policy for internally pricing expected loss. Again, there is need for risk managers to aim at diminishing its high costs. Although risk managers could look more optimistic after quantifying the expected loss but the fact remains that expected loss is not really risk for the bank. Expected loss has priced from the bank. Bank takes protection by provisioning them all. The real risk is uncertain part that is not expected.

Financial institutions could manage expected loss as a cost. When expected loss is calculated at portfolio level, individual expected loss could be basically limited by bank not to exceed this threshold by each obligor. In other words banks could manage expected loss by setting exposure limits according to their default event case.

Literally, regulators bring arrangements for both expected and unexpected losses on portfolio level credit risk. (www.moodysanalytics.com, 2011, Limit Setting and Pricing, pp.6-14)

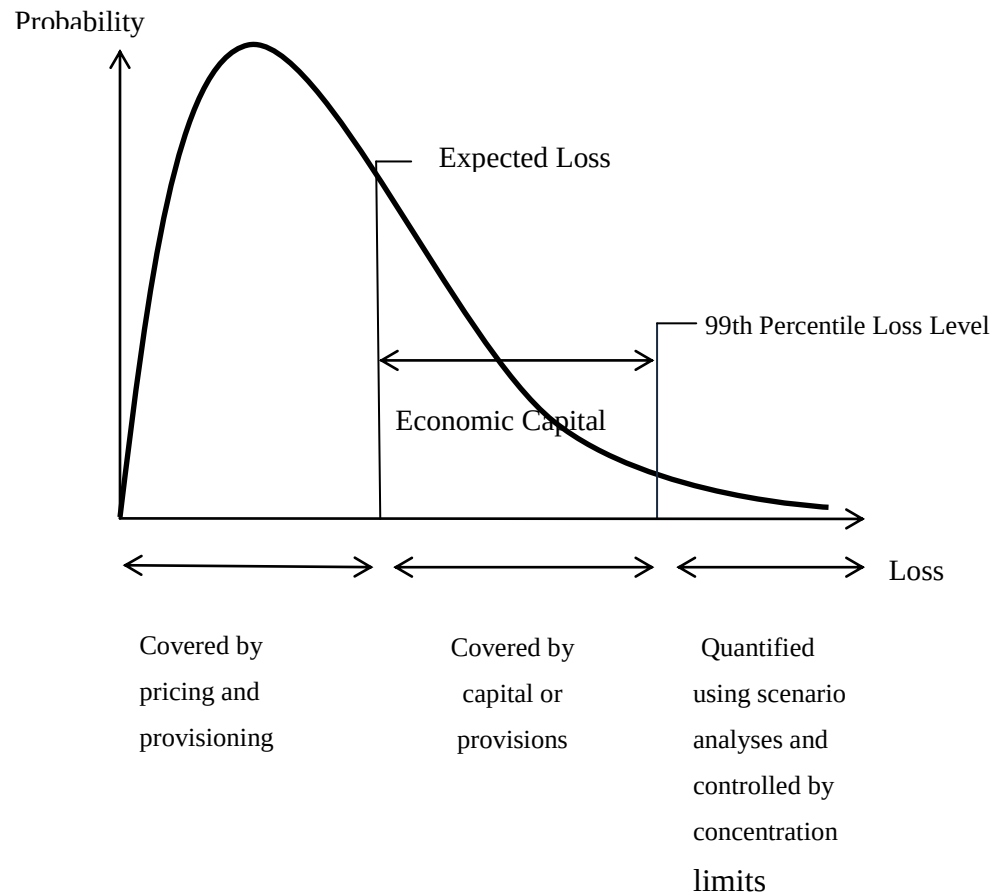


Figure 1.1 Credit Default Loss Distribution

(Credit Suisse Financial Series, 1997, p.25)

At the end of the financial crisis in 2007-2009, the credit risk measurement models have become more comparable to their efficiency. The Causes of the crisis did not only result from inadequate models, but also some reasons that are beyond the models. On the other hand implementation of the model is important to reduce computational cost. Complexity of the model gives significant additional computational cost to institutions.

Credit Portfolio View, CreditRisk⁺ and Moody's KMV models give chance to implement the discriminant analysis on the models (www.aarshmanagement.com, 2013, Modeling Credit Risk – A Survey of Literature, ¶ 1- ¶ 15):

The information known for discriminant analysis is an attempt to classify the successful and failed institutions, which was first implemented by Fitzpatrick, 1932. The method is to accomplish financial ratios so that one could recognize the successful enterprises or vice versa. On this subject matter, Altman, (1968) had earlier advanced these studies, which refers to multivariate discriminant analysis which is generally known as Z-score in credit rating literature. However, the popularity of discriminant analysis diminished in the 1980s. The fact is that there is no precision assumption for its classification.

Maximum likelihood is used as logistic analysis to determine the link between binary responses along with explanatory variables.

(a) It does not assume normality of financial ratios. Instead, it uses a logistic cumulative function to assess the probability of bankruptcy.

(b) It also does not assume the equality of covariance matrices.

In this thesis I will examine in detail the CreditMetrics portfolio model to quantify portfolio credit risk which is proposed by JP Morgan (Gupton et al 1997). A structural approach or option pricing method of KMV as originally proposed by Merton (1974), CreditRisk⁺ of CSFP (1997) and CreditPortfolioView of Mckinsey (Wilson, 1997) also mentioned as portfolio models. Monte Carlo simulation method for portfolio credit risk modeling is investigated in this thesis to take advantage of its complexity and accurate measurement. Monte Carlo techniques were demonstrated with regards to credit risk portfolio modeling.

The sections outline the following; the section 2 explains the credit risk modeling in general. In section 3, the approaches of credit risk modeling are mentioned. In section 4 and 5 CreditMetrics, CreditRisk⁺ framework defined respectively. In section 6 Basel II and new capital accord are mentioned. In section 7 and 8 parameters of credit risk measurement, concentration risk in credit risk modeling are examined respectively. In section 9, 10, 11 Monte Carlo Simulation, simulation steps and outline of simulation steps defined respectively.

Section 12 the results of the application of CreditMetrics is explained and section 13 concludes.

2 *Credit Risk Modeling*

The essential point of the Basel II requirement is classifying the credit risk exposures along with predefined risk weights corresponding to lending activity. Compared to standardized approach in internal ratings based approach, these weights are not predefined, and they are calculated separately from the mathematical models.

Most studies are made to model credit risk from default risk to obligor. General meaning of default risk is that the obligor could not meet the obligation which was stated in her/his commitment. Similarly, default time is not certain for one obligor to model the whole portfolio of credit risk. As well, determination of default time is challenging for the modeler. Therefore finding suitable random time of default is the main component of credit risk models.

Structural approach is one of the main methods for modeling. Detail analysis reveals that structural approach is based on value of the firm which refers to the economical situation of the firm. When one tries to model default event of credit or other components of model, the basic capital structure of the obligor or total value of the assets and default threshold of this value of assets remains the main concepts to analyze.

There are concepts to model credit risk such as structural approach but the requirement or effectiveness of the model differs between banks' portfolios. They are all experimental results to evaluate the choice of models. Thus, there are different conceptual means for modeling approaches to decide model components how to measure.

Economic capital allocation is needed to compensate credit activities for financial institutions. To decide on required capital analytically, probability density function of losses becomes one of the model components.

Credit Loss is calculated for each obligor, whereas, generally portfolio credit loss is the distance between portfolios current value and portfolio future value end of the specified

time horizon. Hence the probability density function of loss determination is the estimation of portfolio future value which is related to modeler approach.

In terms of the current generation of credit risk models, banks make use of either of two conceptual definitions of credit loss, which are default mode and mark to market mode:

Basically the term default mode could be defined as banks that are concerned about only credit defaults or credit does not default. On the other hand mark to market paradigm states both credit movements and default events.

Default mode assumes that if the default event does not occur there is no credit loss faced on the obligor's credit. When a default event is faced, credit loss is calculated from the obligor credit exposure and present value of future recoveries. Current value could be taken as credit exposure but future value differs along with default event whether it is faced or not. If the default event does not occur, the future value measures are as normally from the exposure, otherwise future value measured the one minus loss given default (LGD).

In summary, along with the default mode estimation, the probability function of losses is a joint probability function of:

- Default event occurrence.
- Credit exposure.
- Loss given default rate if default event faced.

As an example, mean and standard deviation of portfolio credit losses is one of the concepts to decide capital allocation. Some institutions credit losses PDF are better explained by the beta distribution in which mean and standard deviation are the parameters.

Generally, the unexpected losses approach is based on the standard deviation of credit losses. Also modeler measures the expected loss along with unexpected loss.

$$\mu = \sum_{i=1}^N EDF_i LEE_i LGD_i$$

LEE_i , Loan equivalent credit exposure.

EDF_i , Expected default frequency.

LGD_i , Loss given default.

Portfolio credit losses standard deviation:

$$\sigma = \sum_1^N \sigma_i \rho_i$$

ρ_i , Correlation between the credit losses

σ_i , Credit standard deviation of losses

Parameter ρ_i , includes the diversification effect of portfolio that result the portfolio standard deviation whether high or not.

$$\sigma_i = LEE_i \sqrt{EDF_i(1 - EDF_i)LGD_i^2 + EDF_iVOL_i^2}$$

VOL , Standard deviation of LGD

All the component of the model increases the reliability of the model output if estimations or assumptions are accurate enough to use (Basel Committee on Banking Supervision, 1999, pp: 13-22).

Table 2.1 Sample of Credit Transition Matrix

Current Credit Rating		AAA	AA	A	BBB	BB	B	CCC	DEFAULT
	AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
	AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
	A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
	BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
	BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
	B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
	CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60

(Gupton, Finger & Bhatia, 1997, p.76.)

Migration table also referred to as the transition matrix probability of migrating from another rating within one year as a percentage. For instance the probability of AA rated loan migrating to BBB rate within one year is 2.16%.

MTM and DM oriented models are both related to credit loss but the comparison between the MTM and DM model do not give the exact idea to implement. DM model is very simple to implement than MTM models. MTM models include putting multi state conditions into analysis. Portfolio culture and model aim is important to choosing MTM model.

So far all credit risk model aims to analyze the probability density function of credit losses at specified time horizon. CreditRisk+, PortfolioManager, CreditPortfolioView and CreditMetrics include the approach implicitly or explicitly.

3 Credit Risk Modeling Approaches

CreditMetrics includes probability density function implicit in the model. Because:

- The model has benefited from computational speed not to have full analytic PDF formula.
- Absence of data because full PDF could not be available for all portfolios.

After model outputs examination there is no standard distribution of probability density function of credit losses. Thus, unlike the market risk models, normal distribution is used as standard distribution in the market. That is the reason why credit risk models need to make more and acceptable assumptions. When market model target quantile range between 95-99%, credit risk models target quantile range 99-99.98%

Categorization of model approaches will be analyzed below (Basel Committee on Banking Supervision, 1999, pp: 28-33):

3.1 Conditional and Unconditional Approach

One of the classifications among models is whether it is conditional or not. In fact all models could be referred to as conditional. In CreditMetrics and CreditRisk+ models do not implicate the business cycle effect. Internal credit ratings do not reflect the cyclical upturn or downturn when that happens. McKinsey and CreditPortfolioView reflect the economic situation at the rating transition matrix during the changes in the credit cycle. Most unconditional models try to reflect the long run averages parameters. Beside in some conditions this approach may mislead the modeler by not reflecting the short run parameters. Correlation is basically more effected from systematical conditions. Conditional model implementation is more complex and results could be overestimated or vice versa. However, determination on model preference is empirical.

3.2 Bottom-Up and Top-Down Approach

Another classification of credit risk aggregation models are termed as bottom up approach or top down approach. Mostly, when banks measure credit risk along with individual asset level on corporate and capital market instruments, the model could be termed as bottom up approach. When the data are analyzed from the consumer credit card or other retail portfolio, top down approach is employed. In fact in model implementation there is no huge difference among them. The differences result from the way of parameter estimations.

In detail, portfolio due to the huge number of exposures, share similar risk within the exposures category such as credit score, same age or location etc. They are classified into buckets and risk measured from the level of these buckets. Default rate and loss given default are calculated from historical data of these segments. Joint default risks are not taken into account for each data in the segments.

In short, the main difference between approaches results from categorization of borrower. After borrower classes are defined, bottom up model implicate that the individual parameters such as bank maps the individual obligor ratings. Hence, the key factor is data reliability and comparability of bank actual portfolio. Otherwise obligor specific effect could be ignored in quantification.

3.3 Structural and Reduced Form Approach

Another concept exists in financial institutions which are structural and reduced form approach for controlling the default or rating transition correlations among the credits.

Structural model as briefly mentioned before, states that customer value of asset level and customer liabilities comparison determines the changes of credit worth along with specified thresholds. By generating random numbers correlation between the migration risk factor is calculated implicitly. CreditMetrics and PortfolioManager model could be given as example for structural models.

Reduced form approaches are used by CreditRisk+ and CreditPortfolioView models. Reduced form models used the functional relationship between obligors as expected default or migration rate. Thus the background factors could be either:

- Observable variables from macroeconomic activity,
- Unobservable random risk factors.

Similarly, model preference is empirical. Besides, when assumptions are made for quantifying correlations the considerations needs to be taken into account:

- Choice of risk factor distributions functions: Gamma, normal etc.
- Whether the correlation structure is correct or stable.

Finally, time horizon could be chosen by the banks:

- Differs between the asset classes along with liquidation period.
- Same for all asset types.

Most banks used one year time horizon for all asset types. Five years or maturity of exposures could be base when choosing time horizons.

These are reasons behind the use of one year time horizon:

- New capital could occur.

- Obligor information could be updated.
- Default frequency may be published.
- Loss mitigation action could be taken as a prevention from future loss.
- Internal budget capital plan and accounting statements could be revealed.

In brief, when it comes to giving the main idea behind the CreditRisk⁺, each loan's probability of default is independent from others, small default probability and independent events reflects the poisson distribution.

Moody's KMV uses the expected default frequency of the firms. Thus, according to KMV default when the value of a firm's assets falls below a threshold level, the firm defaults.

When it comes to Credit Portfolio View, macro economic variables are used to determine default event. Explicit factor that affects the business is the idea behind the default probability.

CreditMetrics framework and parameters will be discussed in detail in next sections.

4 CreditMetrics Framework

CreditMetrics framework starts from standalone risk to joint risk (Gupton, Finger & Bhatia, 1997, pp: 8-15).

As introduced the basic understanding of credit risk is the changes of the worth of debt in a given time horizon. Specifying risk horizon needs to be discussed in one another topic. In this thesis risk horizon refers to 1 year for all obligors.

When it comes to standalone risk, the obligor could stay in the same ratings were it started with that rating to year; the obligor could take into default or obligor rating will upgrade or downgrade.

So we could examine the obligors rating where to go and in which possibility of obligor could take these ratings. When these outcomes are known debt value could be known at the end of year. The point also is that the value changes could differ with the instrument. If we mention about the bond, present value of cash flows could be calculated according to new rating.

So far, standalone and single obligor condition is figured out. When we add single obligor to portfolio, portfolio new value is the point at the year end. Distribution of portfolio value is needed to be determined. Two obligor's rating in which possibility could be taken easily are examined individually. In both, joint portfolio new value in other words joint risk is new item to calculate.

Standalone rating transitions are compatible with the joint possibilities. When obligors are independent, namely correlation among them is zero; joint possibilities are product of standalone obligor's transition probability but in real portfolio the situation is not like this. Correlation must be taken into account. At least they are both under the same macroeconomic factor. Therefore the correlation between rating transition is an issue not to ignore in CreditMetrics portfolio modeling.

Joint Portfolio risk distribution is possible for two of obligor. When it comes to entire portfolio, it is impossible to measure joint likelihoods. The fact is that when 8 ratings from AAA to CCC and default for each obligor, 2 obligor's number of possibilities are $8 * 8 = 64$. When number of obligors are increased 8^N possibilities are the issue to

calculate. Because of this huge growth; large portfolios make use of simulation approach to handle the situation.

5 *CreditRisk⁺ Framework*

CreditRisk⁺ is example for default mode model as distinct from CreditMetrics. Creditrisk⁺ model uses analytical techniques which does not use simulation for estimating credit risk. One could take two outputs default or no default from model.

The methodology used in CreditRisk⁺:

- Distribution and severity of default losses.
- Default event frequency.
- Sectoral analyze.

When estimating portfolio credit risk, model determines default event correlation across default volatility of sectors. Economic factors in countries are usually fundamental reason of defaults. For low probability event effects on credit risk stress test is used.

Default event frequency is modeled on CreditRisk⁺ by using Poisson distribution. Poisson distribution is more advantageous when probability of event comes out rare and there is large number of event.

p_a = Annual probability of default

Expected number of default events of portfolio in one year:

$$\mu = \sum_a p_a$$

Probability of portfolio n defaults in one year :

$$P(n) = \frac{e^{-\mu} \mu^n}{n!}$$

Distribution of number of defaults deduced from poisson distribution because default event is relatively small for each obligor in one year.

Severity of default losses could be obtained after default frequencies are determined. Exposures and concentration of exposures in portfolio and recovery rates are examined for severity of default losses.

Sector analysis is done by estimating volatility of mean default rate. Sector analyze provides the determination of concentration risk and diversification benefits. When model has more sectors to analyze institutions could take benefit from reduced concentration risk.

The loss distribution needs to be calculated. Same losses could be from large losses of single obligors or more small losses in same year. First step to obtain loss distribution is classifying exposures into bands. This step is essential in CreditRisk⁺ which is calculating distribution of losses for whole portfolio and loss given default is classified on band of exposures.

That gives most advantage to avoid huge data on calculation steps when risk manager's tries to implement CreditRisk⁺. This is the strongest side of model in contrast to simulation based studies. (Credit Suisse Financial Series, 1997, pp: 33-36)

When L is unit, Exposure (L_A) :

$$L_A = L * V_A$$

When L is unit, Expected Loss (λ_a):

$$\lambda_a = \varepsilon_a * L$$

V_A and ε_a are the exposure size and expected loss respectively.

6 *Basel II and Basel Capital Accord*

Basel Capital Accord (1998) is replaced by New Basel Capital Accord (2001) which brought new regulations on financial institutions' capital, which deals on how far institutions could be in relation to their capital. Thus, the adequate advanced approach is not realized by Basel Capital Accord (1998) especially on the risk-weighted asset. Basel Committee determined more risk sensitive regulation.

New Basel Capital Accord (2001) includes three additional pillars:

Pillar 1: The Minimum Capital Requirements.

Pillar 2: Supervisory review process.

Pillar 3: Market discipline.

Pillar one reveals that the minimum capital requirement does not modify and the minimum requirement of capital to risk weighted assets which includes operational and market risk is 8%.

There are four key principles of supervisory review process (Basel Committee on Banking Supervision, 2001, pp: 30-31):

Principle 1: This principle emphasizes maintaining bank's capital level by having a process for assessing their overall capital in relation to their risk profile and a strategy.

Principle 2: Review and evaluations must be made by supervisors for banks' internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital ratios. In addition to this supervisors should take appropriate supervisory action if they are not satisfied with the results of this process.

Principle 3: Supervisors should ensure that a bank satisfies the minimum regulatory capital ratios and supervisors have the ability to require banks to hold capital in excess of the minimum.

Principle 4: Supervisors could intervene to prevent capital from falling below the minimum levels required to support the risk characteristics.

Mostly, I will bear upon minimum capital requirements in this study. Pillar 1 mentioned the minimum capital requirements as below:

$$\frac{\text{Bank Capital}}{\text{Risk Weighted Asset}} \geq 8\%$$

Basel II brings that risk weighted assets and includes:

- Credit Risk
- Market Risk
- Operational Risk

According to Basel II, banks has chance to implement the approaches below:

- The Standardized Approach,
- The Foundation Internal Ratings Based approach,
- The Advanced IRB approach.

The main difference between standardized approach, second and third is that it standardized approach reinforces the external credit evaluation to calculate the risk weighted-assets. IRB and advanced IRB approaches give chance to internal risk evaluation for the company.

7 Parameters of Credit Risk Measurement

7.1 Probability of Default

The probability of default is that the obligors could not be able to repay their debt. Most of the banks will use external ratings agencies such as Standard and Poor's or Moody's. Also they would prefer to use their internal PD estimation methods as their own internal rating systems. Commonly, measured credit ratings using corresponding probability of defaults are:

- In the S&P/Fitch rating system, AAA is the best rating. After that AA, A, BBB, BB, B, CCC etc.
- The Moody's ratings are Aaa, Aa, A, Baa, Ba, B, Caa etc.

Probability of default is unavoidable to measure when observing your credit risk. Thus, we could say that measuring default risk involves most of the risk management process. Precisely default risk is an uncertainty for firms prior to their default. We can only calculate one of the most important inputs on loss calculation as a probability assessment that gives us an intuition on how close we are to default. There is no exact way to distinguish firms which will default and those that will not default.

Default event is known typically as rare event. Hence, the disadvantage of measurement of PD involves the variations between firms on their default probabilities.

When one firm defaults, lender suffers from the loss. So, to minimize this risk, generally firms need to pay or spread to compensate for their lender.

As have mentioned in the overview, methods that are mainly used to measure the default probability. First, whatever the approach is, these elements that belong to the firms which provides the determiners to measure the probability of default:

- Firm's asset value
- Firm's asset risk
- Leverage

We could understand that the firm's asset value includes the condition of the firm economic and it's industry from firm's asset value. This value is calculated by the cash flows that were discounted from the appropriate interest rate. It is easy to understand from the case, firm's asset value is not a certain value which incorporates asset risk every time.

Leverage refers to firm's liability. Balance of asset value and firm leverage is important when institution defaults. When an asset value of the firm is under it's leverage, the ability of a repayment obligation could be in danger. Resulting from this, the default point is observed when the firm's asset value is not enough and thus could reach the total liabilities.

When it is comes to calculation of default probability which is from level of distance to default:

$$Distance\ to\ Default = \frac{Market\ Value\ of\ Asset - Default\ Point}{[Market\ Value\ of\ Asset][Asset\ Volatility]}$$

Briefly, probability of default estimation for each borrower is not an easy task. Basically, there are two ways of calculating it. The market value usage and ratings are used to map default probability.

Finding market data is an obstacle sometimes for institutions. Most common approach is using ratings to determine the probability of default. To give the detail for determination of probability of the default using rating system means that the average frequency default of borrower that are in the same credit grade.(Crosbie & Bohn, 2003, pp: 6-9)

As expected, the loss components are exposure at default, loss given default and probability of default. Expected loss classically used to quantify provision to determine profitability. When the risk managers are modeled, credit value at risk, therefore the quantification is beyond expected loss level. The expected loss calculation determines the

underestimation of portfolio credit risk and the risk manager's business decisions that will be corrupted along with the quantification.

The deficiency is not enough to pay attention on grouping defaults and migration. Portfolio models need to quantify the tendency that the obligors default together through correlation factor. Correlation with other key factors EAD, PD and LGD will provide the risk managers the ability to calculate credit portfolio value at risk specified by certain confidence level.

7.1.1 Default Correlation

When obligor defaults, it is assumed that is not the reason only from obligor situation but could be surmised from default correlation between obligors in many cases. Thereby the determination of the default correlation is dependent on the case and interest. After calculation of probability of default between obligors, it could easily be seen that it is not weak enough and strong to focus on.

Correlation between obligors could result in closed relation between firms. In essence therefore while one is a customer to other firms, others could be creditor of other firms as well, since there exist a correlation between them. On the other hand, while the probability of direct relation is lacking, correlation could be seen among the obligors. Thus, they could use the same the raw materials, commodity or sell the same to the same sector.

Explaining the term correlations in detail is beneficial to understand better:

- Joint probability refers to: Obligor-1 and Obligor-2 defaulted before a specified time.
- Conditional default probability refers to: Obligor-1, probability of defaults before a specified time, given that Obligor-2 has defaulted before a specified time

Linear correlation coefficient between the default events From Bayes` rule:

- $p_{b|a} = \frac{p_{ab}}{p_a}$
- $\rho_{ab} = \frac{p_{ab} - p_a p_b}{\sqrt{p_a(1-p_a)p_b(1-p_b)}}$

From the formula one could easily be seen that portfolio joint default correlation not easy task to calculate.

7.1.2 *Correlated Random Variables*

A short look at the basic formulation of correlation reveals that it was the first to generate correlated random variables. Thus our investigation reveals that variables are independent, we could obviously see that the correlation between $A_1, A_2, \dots, A_n$ is zero. Further, formula parameters are covariance and standard deviation:

$$\text{Corr}(A_1, A_2) = \frac{\text{Cov}(A_1, A_2)}{\sigma_{A_1} * \sigma_{A_2}}$$

$$\text{Cov}(A_1, A_2) = E[A_1 A_2] - E[A_1]E[A_2]$$

We could generate covariance matrix of A_i, A_j as $(n \times n)$ matrix. This matrix has specific properties:

- ❖ Symmetric.
- ❖ Diagonal elements are greater than zero.
- ❖ Positive semi-definite.

To remind symmetric positive definite matrix M could be written as:

$$M = U^T D U$$

Where U is an upper triangular matrix and D is a diagonal matrix with positive diagonal elements.

Thereby variance covariance matrix, V is symmetric positive definite we could write

$$\begin{aligned} V &= U^T D U \\ &= U^T \sqrt{D} (\sqrt{D} U) \\ &= (\sqrt{D} U)^T (\sqrt{D} U) \end{aligned}$$

After setting the matrix $C = \sqrt{D} U$,

$C^T C$ is equal to V. This equation is called the Cholesky Decomposition of V. Basel II average asset correlation basis comes from Cholesky Decomposition. (www.columbia.edu, 2010, The monte carlo framework, examples from finance and generating correlated random variables, p.5)

7.1.3 Basel II Average Asset Correlation

There are not many studies on experimental relation on default probability and asset correlation. Lopez (2004) studied empirical relationship between average asset correlation, firm probability of default and firm size. Also Düllmann and Scheule (2003) found relations on them.

Asset value of obligor indirectly could be observed by factor model:

$$r_i = \sqrt{R_i} \phi + \sqrt{1 - R_i} \varepsilon_i$$

r_i , Denotes the asset return of obligor and could be described as that is the variable directly effects in the credit quality change of the obligor.

ϕ , Systematic risk factor that indicates state of economy.

ε_i , Idiosyncratic factor of borrower.

R_i , Percentage of systematic risk.

Obligors under the same conditions are referred to as systematic risk factor. Hence, systematic risk factor will affect the correlation variable. Idiosyncratic factor of borrower is obligor independent condition that will affect the latent variable.

Correlation between obligor i and obligor j formulation is below:

$$Correl(r_i, r_j) = \sqrt{R_i} \sqrt{R_j}$$

Formulation from Basel II, Advanced Internal Ratings Based Approach (A-IRB):

$$R = \alpha * \frac{1 - \exp(-c * PD)}{1 - \exp(-c)} + b * (1 - \frac{1 - \exp(-c * PD)}{1 - \exp(-c)})$$

Variables in formula a, b, c differ according to obligor type. Corporate borrower of coefficients could be taken as $a=0.12$, $b=0.24$, $c=50$. (as cited in Lee, Wang, & Zhang, 2009, pp: 5-6)

When formula is analyzed, the asset correlation shows an exponential decreasing function of PD. As a result, the function built boundary between 12% and 24% correlation for highest and lowest PD of 100% and 0% respectively. Exponential function provides fast decrease on correlation which is determined by the so-called “k-factor” as set at 50 for corporate exposures.

Basel II formula is introduced first for regulatory capital calculation. In industry, the formula was generally accepted for credit risk calculation especially for calculation of required economic capital.

So far, the explanations are pointed to the fact that the asset correlations show to what extent the asset value of obligor is in line with the asset value of another borrower.

In the same manner, correlation according to model is introduced as the dependence of the asset value of a borrower on the general state of the economy. This definition proposed that all borrowers are linked to each other by this single risk factor ($\emptyset$).

The asset correlations used in model is calculated through supervisory formulas and estimations of asset correlations provided by Internal Ratings Based Approach framework. According to Basel Committee on Banking Supervision (2005, p.12) G10 supervisors’ time series of corporate default data reveals that:

(1) Asset correlations decrease with increasing PDs. This is based on both empirical evidence and intuition. Intuitively, for instance, the effect can be explained as follows: the higher the PD, the higher the idiosyncratic (individual) risk components of a borrower. The default risk depends less on the overall state of the economy and more on individual risk drivers.

(2) Asset correlations increase with firm size. Again, this is based on both empirical evidence and intuition. Although empirical evidence in this area is not completely conclusive, intuitively, the larger a firm, the higher its dependency upon the overall state of the economy, and vice versa. Smaller firms are more likely to default for idiosyncratic reasons.

7.2 *Loss Given Default*

Main causes of credit losses events are:

- Change in LGD value,
- Change in credit worthiness,
- Change in credit spread on MTM models,
- Change in bank exposure.

When obligor defaults, one of the components of credit loss calculation is loss given default. LGD could be referred to as one minus recovery rate.

Credit Loss is calculated by:

$$\text{Credit Loss} = \text{LGD} * \text{PD} * \text{EAD}$$

Under the IRB approach, banks may use their internal experience but the fact is that some common information is known according to academic and industry studies. Categorized information is used by supervisors to determine whether banks approach is adequate enough. The Wharton Institution Center (2001, p.4) studies about the characteristic information which draws attention below:

1. Most of time recovery as a percentage of exposure is either relatively high or low. The recovery or loss distribution is said to be “bimodal”. Hence thinking about an “average” recovery or loss given default can be very misleading.
2. The most important determinants of which mode a defaulted claim is likely to fall into is whether or not it has secured its place in the capital structure of the obligor (the degree to which the claim is subordinated). Thus bank loans, being at the top of the capital structure, typically have higher recovery than bonds.
3. Recoveries are systematically lower in recessions and the difference can be dramatic: about one-third lower. That is losses are higher in recessions, lower otherwise.

4. Industry of the obligor seems to matter: tangible asset-intensive industries, especially utilities, have higher recovery rates than service sector firms, with some exceptions such as high tech and telecom.
5. Size of exposure seems to have no strong effect on losses.

Default event could be seen in variant conditions according to BIS (as cited in Schuermann, 2001, p.5):

- Mostly one thinks when obligor defaults, borrower could not pay its debt.
- When obligor could not pay 90 days later from due date, that means that the obligor has accepted to fall into default.
- Obligor could be applied in conditions of bankruptcy.

We know that in credit risk models, default events are treated as full loss while recovery rate is in progress but in real life, most defaults results to only delay and recovery rate is 100% so that such default events could be observed differently in credit risk models.

As I mentioned indirectly before loss is obtained from ratio, LGD to exposure. This losses include principle loss, non-performing loan carriage cost and workout expenses.

Moreover, Basel II provides that banks could estimate essential elements to measure LGD of credit risk. Reliability of LGD estimation mostly depend on that bank has enough and suitable data with regard to its history and experiences.

One very important point that is needed not to be skipped is that there is a positive correlation between the whole default rate and loss given default.

Modeling LGD could be a very sophisticated process for institutions thus needed another study for detail analysis of each. Briefly, basic regression, advance regression, neural networks, tree methods, machine learning could be used to model LGD. Apart from the basic regression, other methods which are sophisticated models nevertheless, they are better fit in the data. Moreover basic regression is easier and score card could be produced after implementing this model.

LGD is not a binomial data like PD. Their variables are widely fit beta distribution and are similar to continuous variable.

Obstacle of LGD modeling institutions have not enough data to use, in fact their collection of data started in 1996 after regulators arranged that institutions become submissive for internal rating bands. In contrast to PD modeling, LGD value was for several years made clear after default events of each obligor.

According to investigations and findings LDG is an abnormal variable hence, linear regression was used to get standard normal target variable to estimate LGD, Gupton and Stein (2002). However the challenge in the inverse transformation on of target variable is as a result to of large errors in getting the actual number. Therefore, the purpose is to minimize standard error of actual target and not necessarily to transform numbers.

Again, other notable indication of LGD modeling challenge is knowledge for collateral loans. When collaterals are included in the model, there is a linear relationship between LGD and collateral. However the data often shows that the LGD dispersion over collateral value is not linear. Therefore, disappointment could be observed when the expectations were not satisfied by the modelers, Grunet and Weber (2007). In these situations, the evidence approach is useful in incorporation of business expectation and data statistical strengths. (as cited in Yang & Tkachenko, 2012, pp:81-83)

7.3 Exposure at Default

When looking at the modeling parameters such as PD, LGD and EAD, PD estimation seems to have supposedly developed and the most used of modeling in the market. Thus, we examine and analyze practiced models for PD in the business. However, LGD and Exposure at default are not ripe for estimation. Along with the Basel accord, parameter estimation is unavoidable as the reliability of these parameters reflects a model results on how much model is close to accuracy.

Exposure at default is an amount that needs to be paid after the default event has occurred. Generally, this amount complies with outstanding balance. Notwithstanding that some products have limited usage behind the outstanding balance, so there is a possibility of increment of exposure from the date credit was used and default event was faced.

The formula for modeling EAD is below:

$$EAD = bal_0 + conversion\ factor * (auth_0 - bal_0)$$

bal_0 : Risk that is outstanding amount at current time,

$auth_0$: Risk that is authorized amount at current time,

$undrawn_0$: Undrawn amount at current time $(auth_0 - bal_0)$.

The formula demonstrates that EAD could be measured by existing risk which has already occurred in addition to extra potential risk between limit and outstanding amount percentage. This percentage is used as a conversion factor which refers to an amount obligor that could use before default event as well as estimating challenging factors could be

$$Conversion\ factor = \frac{\max(bal_1 - bal_0, 0)}{undrawn_0} \quad if\ undrawn_0 > 0$$

$$0 \quad if\ undrawn_0 \leq 0$$

bal_1 : Risk that is outstanding amount at default time,

Conversion factor modeling could vary and this topic could deeply be analyzed.

In brief, a model was chosen to calculate EAD factor not as directly outstanding amount at default. (Yang & Tkachenko, 2012, pp: 83-85)

7.4 Credit Spreads

While, credit spread is one of the key inputs on MTM models. Notwithstanding the fact that measuring spread between yields are very difficult. Most MTM models sets fixed spread before executing the model.

8 Concentration Risk

Experiences have shown that from history, concentrations on credits are another significant effect on Banks failure. Credit concentrations risk is not only for the Banks but also individual firms as well. When large and unstable exposure size credits are given to homogeneous asset type, the entire banking system is undermined in this situation.

Often examples are made from US Banks such as Enron, Worldcom, Parmalat and in a number of banks when they face huge losses from their creditors. On the other hand, one of the fact is for the banks to have a significant concentration on a single sector for example energy sector is because the largest exposures are given mostly on regional needs that productivity mostly depends on it.

Works on concentration risk has earlier been carried out by group of researchers from the Research Task Force of the Basel Committee on banking supervision. Their studies were neither to give any recommendation to the institutions. Actually, sector concentration is more challenging to measure and understandable than single named concentration because of the lack of information given to lead institutions from authorities. As a result of the fact that the studies are not just to give specific recommendations to institutions, they however, demonstrate which methodology implementation is suitable to their portfolios to deal with concentration risk. (Basel Committee on Banking Supervision, 2006, p.3)

Generally speaking, management of concentration risk differs across banks. Mostly, exposure limit system which differs from bank to bank according to their strategic goals and internal economic capital models are used to measure obligor's risk contributions which are examined in best practices banks. Incorporating concentration risk in pricing new loans is observed nevertheless.

In relation to the best practices, banks are observed in detail on how they implement the items mentioned above: For most banks methodological approaches are not equal. Some prefer to group exposures to risky obligors while, some prefer to implement Herfindahl-Hirschman index etc.

8.1 *Measuring Credit Concentrations*

Measuring concentration risk approach differs when ones look at the literature. An attempt to reduce the concentration risk over the portfolio is monitoring the concentration index. Particularly the index is used for portfolio on single measure and the most common index is the Herfindahl-Hirschman Index (HHI).

Models could not observe credit concentrations with just measurement of Value at risk. The term stand-alone credit value at risk or absolute loss means that the portfolio includes only exposures belonging to one obligor. When we have this quantification, then name level VAR divided by absolute loss. This ratio is useful to compare obligors which are highest concentration and the lowest.

Risk managers could have idea by the quantifications which are needed to most of the monitor and hedge which are not. Hedging benefit gives option for managers to choose an efficient portfolio and also gives option to choose the lower cost portfolio to institution.

When risk managers want to decide which obligor gives concentration and which obligor gives diversification effect. They could also use their concentration index along with this formula:

$$\frac{(CVAR_i/ABSL_i)}{\sum_{j=1}^n CVAR_j / \sum_{j=1}^n ABSL_j}$$

CVAR denotes credit value at risk, ABSL denoted absolute loss, n denote obligor number

This formula provides more comparable results between obligors which is more standardized version of the ratio. When index is more than 1, risk managers could think that obligor add concentration effect. When index less than 1 vice versa.

This information is hidden when risk managers only examines value at risk. After these calculations they could reevaluate the portfolio whether they will add new business to their portfolio or not. As a result of this, the obligor could be seen as a good business partner to add in portfolio but it may not be after this calculation. (Reynolds, 2009, pp: 3-6)

9 Monte Carlo Simulation

9.1 Building Simulation Framework

The most commonly accepted quantifying portfolio credit risk measure of standard deviation is not adequate in approach. Easily observable fact is that the credit risk distribution is not normal and thus, typically asymmetrical. As a solution alternative approach, the assumptions possible portfolio scenarios are generated randomly along with simulation. After scenario generation, calculation of the desired percentile level is more accessible than analytical calculating method.

Monte Carlo simulation methodology better describes the distribution of portfolio values in contrast to analytical method. These are the steps for building a simulation framework below (Gupton, Finger & Bhatia, 1997, p.113):

- Scenario Generation,
- Establish thresholds for obligor in portfolio,
- Generate scenarios according to the normal distribution,
- Map the scenarios to credit rating scenarios,
- Calculating credit losses for each trial,
- Calculating portfolio losses for each trial,
- Statistical results.

Credit events are triggered by the movements underlying unobserved latent variables. Thus, there is an assumption that this variable depends on external “risk factors”. Common dependence on the same risk factors gives rise to correlations in credit events across obligors. Detail calculation steps are explained in a qualitative manner. Overview of Monte Carlo Simulation models are defined below (as cited in Brereton, Kroese & Chan, 2012, pp: 5-17):

9.2 Factor Model

Factor model generates random variables. When these random variables exceed threshold from transition matrix, component of portfolio goes into default:

$$Default(D_i) = (X_i > p_i), \quad i = 1, \dots, n$$

Obligor i defaults when random number X crosses threshold

If a model has one factor, it could be called a single factor model. If it includes more than one factor, then it is referred to as multifactor model. These factors correspond to macroeconomic and industry based factors.

Gaussian factor model:

$$X_i = \alpha_{i1}Z_1 + \dots + \alpha_{in}Z_n + \alpha_i\varepsilon_i$$

Each Z_i, ε_i standard normal random variable as a result of this distribution of X is standard normal.

The algorithm of factor model:

1. Generating random variables: They are factors systematic for all the component of portfolio defendants and the factors which are component specific factors are that means only related to component.
2. Calculating the $X_1, X_2, \dots, X_n$ which are functions of the factors for model
3. Finally calculating the Losses

9.3 Copula Models

Fundamental differences between models are describing the dependency structure. Copulas are very common in credit risk models to express dependency. X random variables comes from uniform random variables $U_1, U_2, \dots, U_n$ that is the main difference from factor model and they could be converted to random variables $X_1, X_2, \dots, X_n$ by choosing distributions. [Nelson, 2006]

$$X_i = F^{-1}(U_k)$$

The Gaussian Copula is the form (Li, 2000):

$\Phi_r(\cdot)$ is multivariate normal distribution function mean vector 0 and correlation matrix Γ .

$$C_G(u_1, u_2, \dots, u_n) = \Phi_r(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \dots, \Phi^{-1}(u_n))$$

10 Monte Carlo Simulation Steps

Here I employed CreditMetrics framework based on “Migration Analyses” and “Default Event”. Simulation program results mainly shown for default event analysis is more accepted in industries because of its less calibration difficulty.

Program supports the extended version of default event analysis. That could be mentioned as full transition model which requires a rating transition matrix, threshold matrix and rating transition matrix. Rating transition threshold matrix is derivated from implementing inverse normal distribution function of probability of obligor’s rating transitions.

When it comes to simulation program logic, user needs to choose spread before program run. When user chooses spread is equal to zero. Program decides to run to analyze only default event. Otherwise by giving a credit spread, program could estimate a distribution over the change in mark-to market value to portfolio credit risk.

Loss calculation is essential after mode selection on simulation program. In the binary mode, in the event of default, program assumes that loss is a fraction of the face value derived through obligor’s LGD, while the loan retains its book value in non-default state. This allows recognizing idiosyncratic risk in the recoveries.

Simulation program provides:

- Single obligor based run.
- Sectoral based run.

The main difference among the correlation factor will decide whether program will use:

- Sectoral correlation,
- Formula based single obligor correlation.

Mainly formula based on correlation used to analyze a result which is taken from Basel II below as detail information mentioned before.

$$Rho = 0.12 * \frac{1 - \text{Exp}(-50 * PD)}{1 - \text{Exp}(-50)} + 0.24 * (1 - \frac{1 - \text{Exp}(-50 * PD)}{1 - \text{Exp}(-50)})$$

Asset correlation and probability of default are driven factors in credit risk modeling. Commonly default correlation in credit risk models are modeled by building linkage between asset correlation and default probability which is efficient in credit loss calculation.

10.1 Analysis of Default Event

The user types zero for spread program determines run on default mode as mentioned before. Besides, “latent variable” plays a vital role in the determination. This variable calculated per obligor are most of things done for the sake of production of that latent variable.

Recall the formula:

$$l_i = w_i * y_i + w_j * \epsilon_i$$

“ w_i ” is the degree of the obligor’s exposure to the systematic risk factor, “ y ” is systematic risk factor, “ ϵ_i ” is idiosyncratic risk factor of the obligor

After calculation of latent variable, latent variable will be compared against the derived cut-off value (default threshold) and rating transition threshold which is specific to each customer:

- When the latent variable falls under the cut of value for an obligor, it defaults.
- When the latent variable is greater than rating transition thresholds it would take relative new rating from transition table.
- Otherwise obligor’s rating remains the same.

10.2 Generating Correlated Random Numbers

Random numbers are trigger factor for generating scenarios in simulation. Latent variable components are systematic and idiosyncratic risk factors. All obligor in portfolio effected from systematic factors and idiosyncratic risk factor. Idiosyncratic risk factor is independent indicator of obligor situation. The related code part below is the summarized version to highlight the random number generation:

```
for trial=drange(1:trialNumber)

systematic(trial,1) = Randoms{...};

for credit=drange(1:numberOfCredits)

    idiosyncretic = Randoms(...);

if strcmp(correType,'Single')

pd=pdFinder(Portfolio,ratingNo);

rho=calcRho(pd);

creditSim = idiosyncretic{1,1} * square(minus(1,power( rho, 2))) + systematic(trial,1) * rho;
```

Systematic random variable was generated first to affect all credits in simulation. Then idiosyncratic random number was generated for each credit. The point is to generate *creditSim* variable. Recall asset values of obligor were indirectly observed by the factor model implementing this formulation.

$$r_i = \sqrt{R_i} \phi + \sqrt{1 - R_i} \varepsilon_i$$

creditSim in latent variable is a simulation to determine the obligor's new rating, whether credit worth deteriorate or increase or remain the same.

10.3 Loss Calculation Logic

In simulation program loss calculation:

When obligor defaults, program returns new rating as zero.

Therefore:

$(\text{ratingNo} - \text{newRatingNo}) == \text{ratingNo}$ case is validated when obligor defaults.

Loss will be calculated by multiplying exposure at default and loss given default

$\text{creditLoss} = -\text{LGD} * \text{EAD}$

Otherwise spread is multiplier in loss calculation

$\text{creditLoss} = ((\text{ratingNo} - \text{newRatingNo}) * \text{spread}) * \text{EAD}$

Overall related part below:

if $((\text{ratingNo} - \text{newRatingNo}) == \text{ratingNo})$

$\text{creditLoss} = -\text{Portfolio}\{\text{minus}((\text{Row} + \text{credit}), 1), 4\} * \text{Portfolio}\{\text{minus}((\text{Row} + \text{credit}), 1), 2\};$

else

$\text{creditLoss} = ((\text{ratingNo} - \text{newRatingNo}) * \text{spread}) * \text{Portfolio}\{\text{minus}((\text{Row} + \text{credit}), 1), 2\};$

end

10.4 Expected Loss Unexpected Loss and Alpha

I used confidence level 99.9% referred to as alpha in simulation. Credit loss is calculated along with the obligor's new rating. The algorithm for expected loss calculation was described below.

For each trial credit loss will be calculated for each obligor. All credit losses were hold in matrix then gathered for trying to find portfolio loss of trial 1. This process was the same for all trials. The essence is to allow each to get portfolio losses for each trial.

for trial=drange(1:trialNumber)

for credit=drange(1:numberOfCredits)

...

lossMatrix (trial,credit) = -creditLoss;

```
loss = loss + creditLoss;
```

```
end
```

```
losses(trial, 1) = -loss;
```

```
end
```

Expected loss is equivalent to portfolio losses mean. While Portfolio loss mean is found below:

```
portmean = 0;
```

```
for simNo=drange(1:trialNumber)
```

```
portmean = portmean + PortfolioLoss(simNo, 1);
```

```
end
```

```
portmean = portmean / trialNumber;
```

```
eL = portmean;
```

All portfolio losses were sorted in descending order:

```
PortfolioLoss=sort(losses, 'descend');
```

I mentioned that the portfolio losses are sorted in descending order in code above. Basically, is to find unexpected loss. Alpha is the parameter for calculation of unexpected loss because unexpected loss is above the mean loss. Thus, it is calculated as a standard deviation from the mean or expected loss at a certain confidence level referred to as alpha. On the other hand, unexpected loss corresponds to “Credit VaR” as the essential output of credit simulation.

Programs which get the UL from descending sorted portfolio losses correspond to (Trial number * (1 – alpha))th element of the portfolio losses.

```
uL = PortfolioLoss(round(trialNumber * minus(1,alpha)), 1);
```

10.5 Trial Number

Optimal trial number is important for the power of results in simulation. In CreditMetrics Monte Carlo Simulation Model standard error will be differing for different trial number. This number could be tested whether they are optimal enough for the portfolio or not. . Increasing the trial number enough, guarantees the power of Monte Carlo Analyze. I employ simulation for 25000 times.

10.6 Ratings Definitions, Rating Transitions and Default Rates

Rating agencies Moody's, S&P and Fitch two types of investment grade and speculative grade rating definitions and relative ratings are below:

Table 10.1 Moody's, S&P and Fitch Rating Definitions

Definition	Moody's	S&P	Fitch
Investment Grade			
Prime, Maximum Safety	Aaa	AAA	AAA
Very high grade quality	Aa1	AA+	AA+
Very high grade quality	Aa2	AA	AA
Very high grade quality	Aa3	AA-	AA-
Upper medium quality	A1	A+	A+
Upper medium quality	A2	A	A
Upper medium quality	A3	A-	A-
Lower medium grade	Baa1	BBB+	BBB+
Lower medium grade	Baa2	BBB	BBB
Lower medium grade	Baa3	BBB-	BBB-
Definition	Moody's	S&P	Fitch
Speculative Grade			
Speculative	Ba1	BB+	BB+
Speculative	Ba2	BB	BB
Speculative	Ba3	BB-	BB-
Highly Speculative	B1	B+	B+

Highly Speculative	B2	B	B
Highly Speculative	B3	B-	B-
Substantial risk	Caa1	CCC+	CCC+
In poor standing	Caa2	CCC	CCC
In poor standing	Caa3	CCC-	CCC-
Extremely speculative	Ca	CC	CC
Maybe in or extremely close to default	C	C+,C,C-	C+,C,C-
Default		D	D

(www.multiple-markets.com, 2013, The Ratings Table)

I used Standard and Poor's 2012 annual global corporate defaults and rating transitions for each obligor. The S&P Data source is from 1981 to 2012 (for details see Appendix)

Table 10.2 Global One-Year Relative Corporate Transition Rates (1981-2012)

From/To	AAA	AA	A	BBB	BB	B	CCC/C	D	NR
AAA	87.17	8.69	0.54	0.05	0.08	0.03	0.05	0.00	3.38
	(9.11)	(9.13)	(0.86)	(0.31)	(0.25)	(0.20)	(0.40)	(0.00)	(2.66)
AA	0.54	86.29	8.36	0.57	0.06	0.08	0.02	0.02	4.05
	(0.55)	(4.90)	(3.99)	(0.75)	(0.25)	(0.24)	(0.07)	(0.07)	(1.91)
A	0.03	1.86	87.26	5.53	0.36	0.15	0.02	0.07	4.71
	(0.13)	(1.15)	(3.47)	(2.10)	(0.49)	(0.35)	(0.07)	(0.11)	(1.91)
BBB	0.01	0.12	3.54	85.09	3.88	0.61	0.14	0.22	6.39
	(0.06)	(0.23)	(2.31)	(4.62)	(1.82)	(1.02)	(0.24)	(0.26)	(1.79)
BB	0.02	0.04	0.15	5.18	76.12	7.20	0.72	0.86	9.71
	(0.06)	(0.16)	(0.39)	(2.35)	(5.02)	(4.63)	(0.92)	(1.04)	(2.84)

B	0.00	0.03	0.11	0.23	5.42	73.84	4.40	4.28	11.68
	(0.00)	(0.13)	(0.37)	(0.33)	(2.50)	(5.30)	(2.52)	(3.32)	(2.98)
CCC/C	0.00	0.00	0.16	0.24	0.73	13.69	43.89	26.85	14.43
	(0.00)	(0.00)	(0.70)	(1.01)	(1.29)	(8.42)	(12.62)	(12.48)	(7.19)

(Standard & Poors Global Credit Portal, 2013, p:31-32)

The NR column which means Standard & Poor's is no longer rated and an obligor faces default events. They are included in such a way that the obligors default sample of one year's default is fully observed. The probability of rated defaulters and of no longer defaulters is combined as one column in simulation.

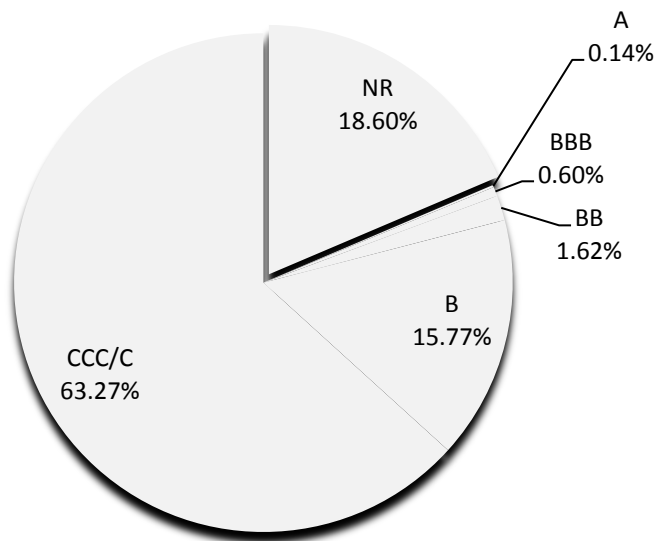


Figure 10.1 Default Distribution by Rating prior to 'D' (1981-2012)

(Standard & Poors Global Credit Portal, 2013, p.13)

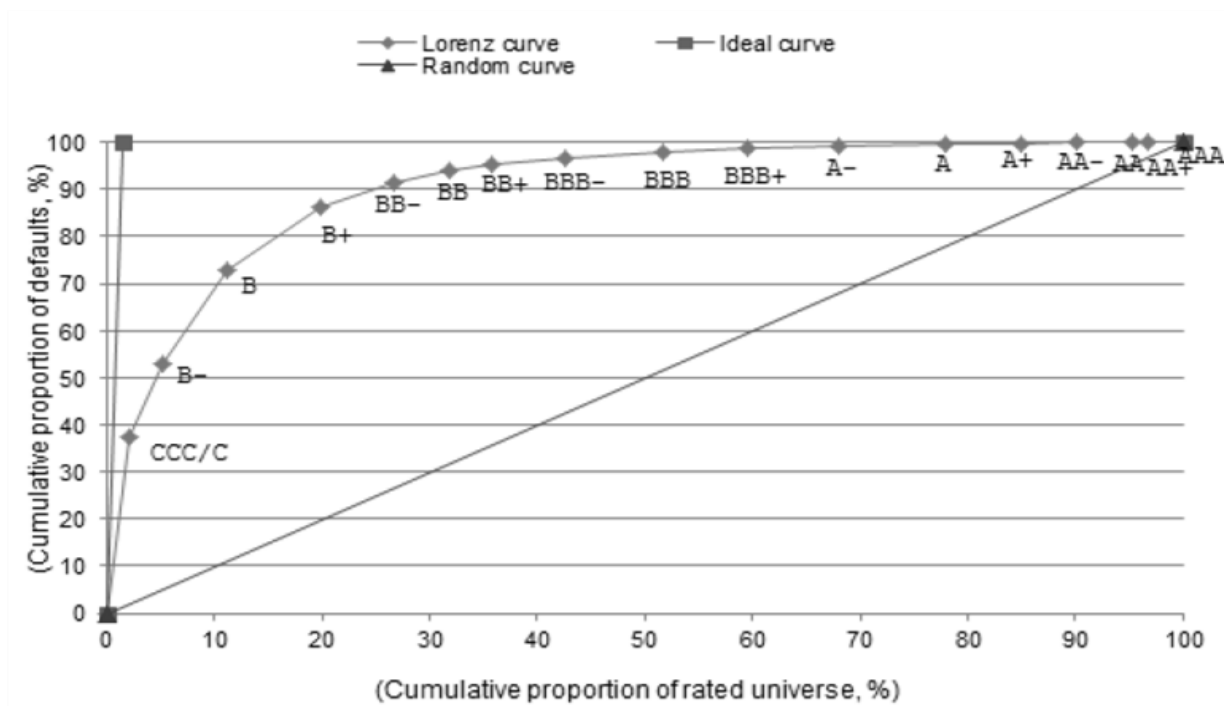


Figure 10.2 Global One-Year Relative Corporate Ratings Performance (1981-2012)

(Standard & Poors Global Credit Portal, 2013, p.38)

The area between Lorenz Curve and random curve refers to Gini coefficient. The global average of one year transition to default has a year gini coefficient of 82.2% which shows the power of analysis.

10.7 Credit Unexpected Loss

Credit unexpected loss means credit contribution to portfolio VAR. The calculation logic is described below:

Portfolio losses are sorted out to find portfolio VAR. Before that trial numbers and portfolio losses are copied not to lose the original trial numbers of portfolio losses. Trial numbers or scenario numbers are held when it exceeds the EL level. Credit loss contribution to portfolio loss which is beyond EL level is aggregated for each obligor. Thereby, when this ratio is multiplied by the portfolio unexpected loss, credit unexpected loss achieved.

$$\text{Credit}_i \text{ Unexpected Loss} = \text{Portfolio UL} * \frac{\text{Credit}_i \text{ losses beyond EL level}}{\text{Portfolio losses beyond EL level}}$$

10.8 Equally Exposure Distribution to Portfolio

In simulation program, credit losses are recalculated in the same scenario as if all obligor has same exposure amount that is equal to an average total portfolio exposure. Thereby new expected loss and unexpected loss amount are calculated as if total the portfolio exposure is divided equally to all obligors. In the same scenarios we could get two unexpected losses which are comparable. The difference between results could be meaningful for credit risk managers.

11 Outline of Monte Carlo Simulation Steps

The inputs of Monte Carlo simulation:

- Obligor loan.
 - Obligor rating.
 - Probability of rating transitions and default.
 - Rating transitions and default thresholds.
 - Loss given default.
 - Correlation type.
 - a) Single based correlation from Basel II formula,
 - b) Sectoral based correlation.
 - Obligor sector.
 - Confidence interval.
 - Trial Number.
 - Spread.
1. Systematic risk factor generated at random which is a standard normal distribution.
 2. If a single option is chosen for simulation, for each obligor:
 - a. Idiosyncratic factor is generated.
 - b. Probability of defaults taken from transition matrix and obligor's correlation is calculated (ρ). (Formula is function of PD: If obligor has high PD, obligor is low correlated to economy; if obligor has low PD, obligor is high correlated to economy.)
 - c. Model needs random numbers to decide new rating. creditSim is a random number which is a function of ρ , random idiosyncratic and systematic factor. So we have a random number which has idiosyncratic and systematic scenario and correlation effect.

$$\text{creditSim} = \text{idiosyncratic} * \text{Sqr}(1 - \rho^2) + \text{systematic} * \rho$$

If sectoral option is chosen from simulation:

- a. Idiosyncratic factor is generated.

- b. sectorRho is taken by a function which is filled in excel as sectoral correlation.
- c. creditSim is a function of sectorRho, random idiosyncretic and systematic factor.

$$\text{creditSim} = \text{idiosyncretic} * \text{Sqr}(1 - \text{sectorRho}^2) + \text{systematic} * \text{sectorRho}$$

3. When simulation runs in default mode by typing spread as zero, each credit loss is calculated by multiplying the loss given default and exposure.
4. When simulation does not run in default mode (rating transition mode), each credit losses are calculated by multiplying loss given default, exposure and spread.
5. Each credit loss is held in a Matrix.
6. Total loss calculated for each trial as portfolio losses which are held in a Matrix.
7. Expected loss is equal to mean of portfolio losses which are calculated from each trial.
8. Unexpected loss is found from a given percentile.
9. Expected Loss and Unexpected Loss is found for each credit.
 - a. Credit expected loss is found by multiplying exposure, LGD and PD of credit.
 - b. Credit unexpected loss is found by multiplying portfolio unexpected loss, credit loss contribution to portfolio loss that exceeds the portfolio unexpected loss level.
10. Loss calculation repeats for each obligor as if all obligor has exposure amount equals to the average of portfolio amount.
11. Expected and unexpected loss is found as if all obligor has exposure amount equals to the average of portfolio amount compared to the real expected and unexpected loss.
12. Finally total portfolio exposure, average portfolio exposure amount, percentage of expected loss in portfolio, percentage of unexpected loss in portfolio, percentage of credit expected loss and unexpected loss to exposure is calculated.

12 Property of Portfolio and Simulation Results

Hypothetical corporate portfolio that includes a number of 500 obligors was used. Each customer defined by exposure, rating and LGD attributes. The S&P rating classes was used. Total portfolio amount 7,651,455,672 TL. Minimum exposure is 2,085,657 TL. Maximum exposure is 195,437,820 TL. Portfolio average is 15,302,911 TL. Figures give an idea of portfolio distributions below:

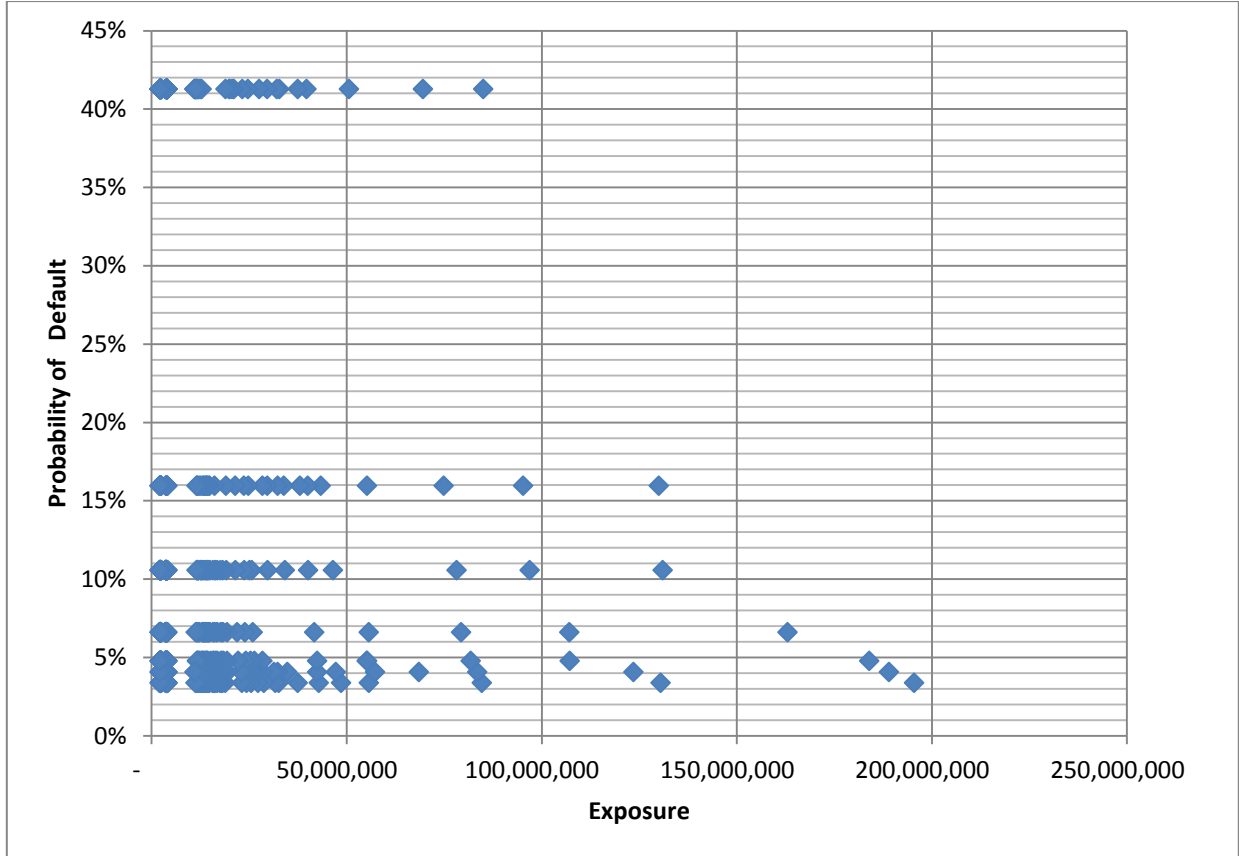


Figure 12.1 Exposures and Probability of Defaults

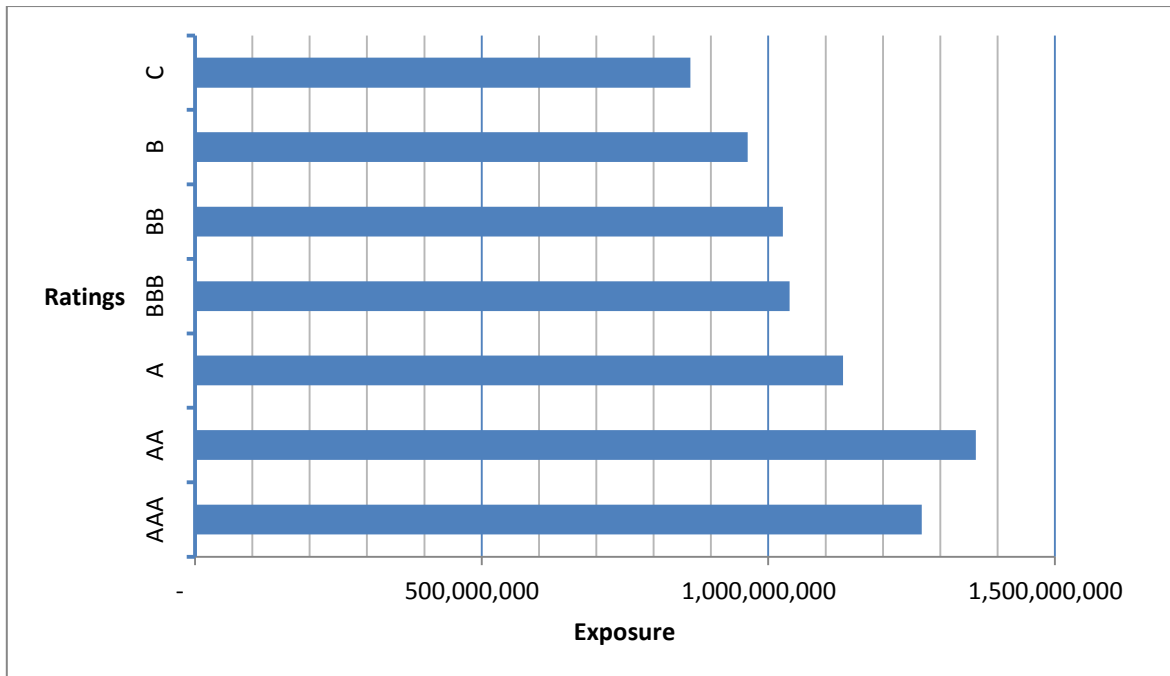


Figure 12.2 Rating Distribution of Portfolio

I simulate the portfolio for 25000 trials 10 times.

One of simulation output is portfolio expected loss which is around 384,000,000 TL

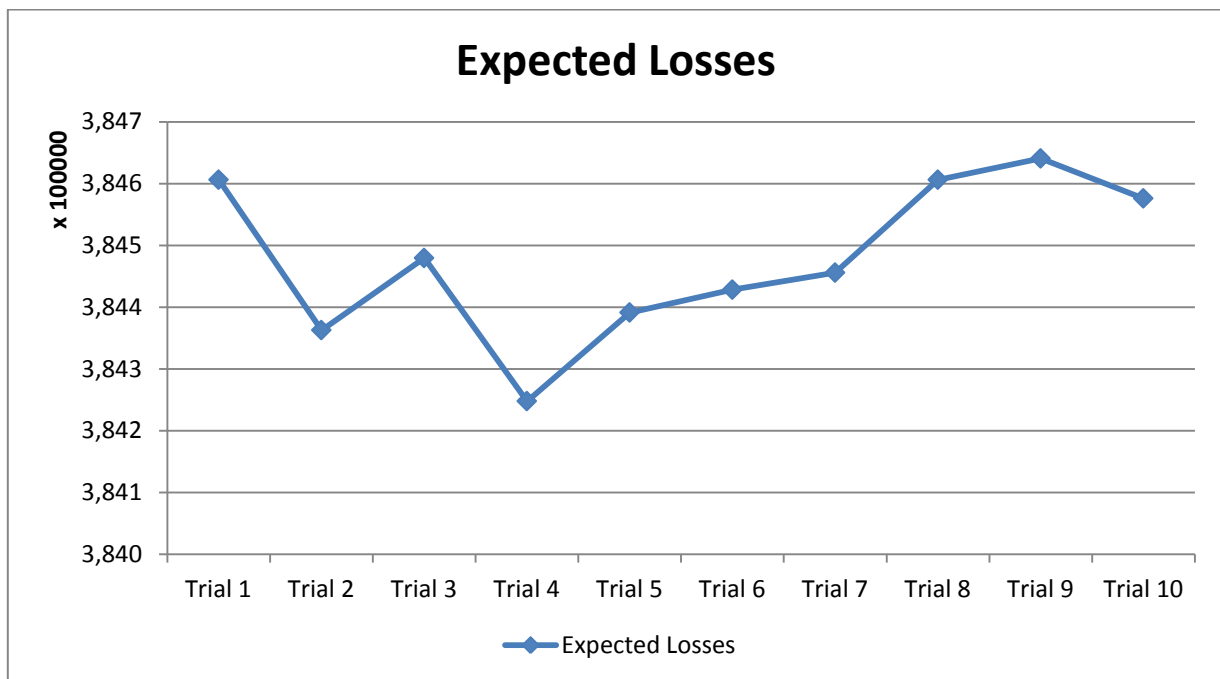


Figure 12.3 Expected Losses Distribution of Portfolio

Secondly, distribution shown for unexpected loss:

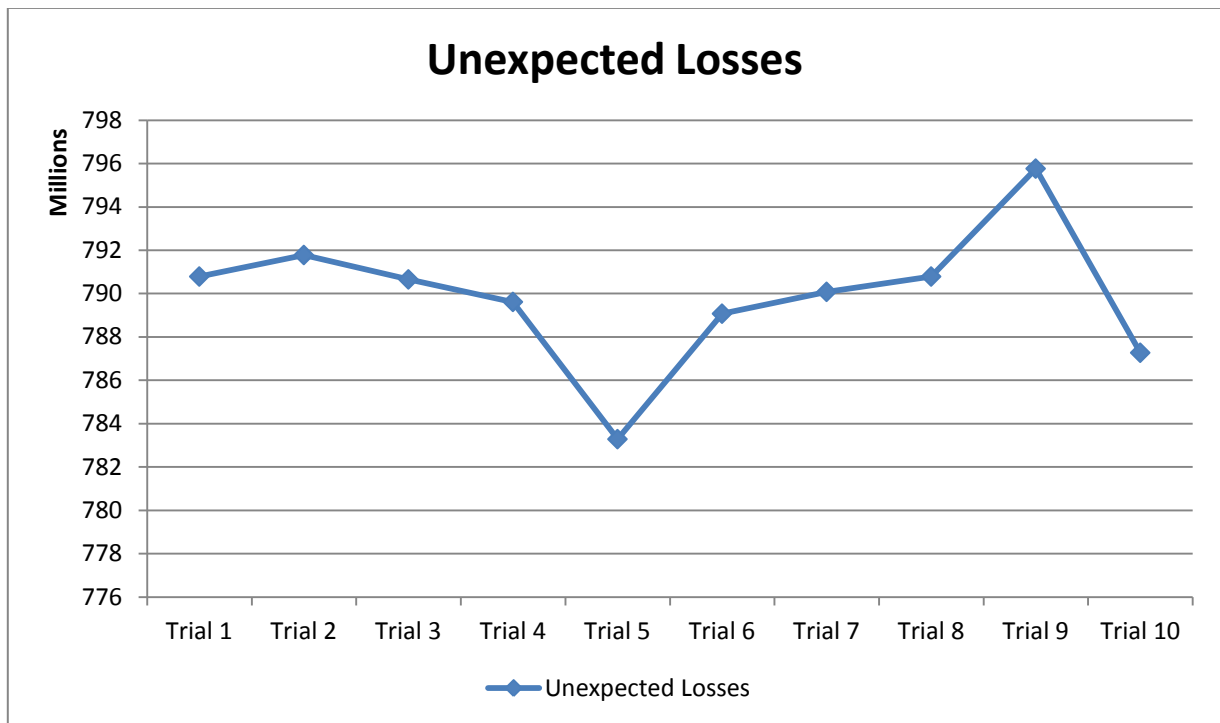


Figure 12.4 Unexpected Losses of Portfolio

Thirdly, expected loss of equally exposure distributed portfolio:

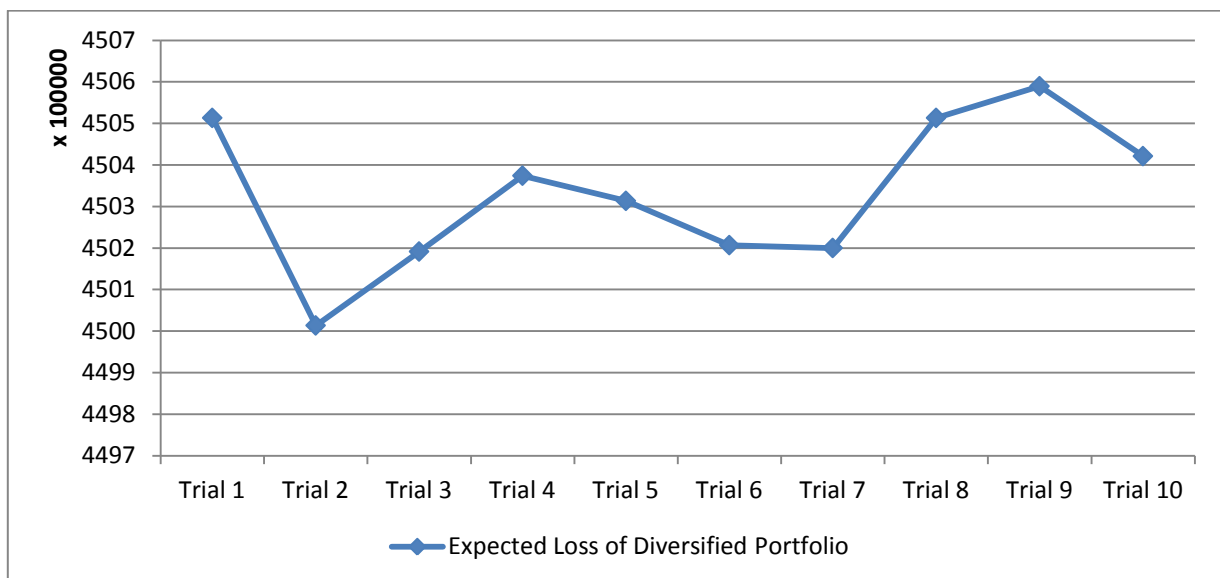


Figure 12.5 Expected Losses of Equally Exposure Distributed Portfolio

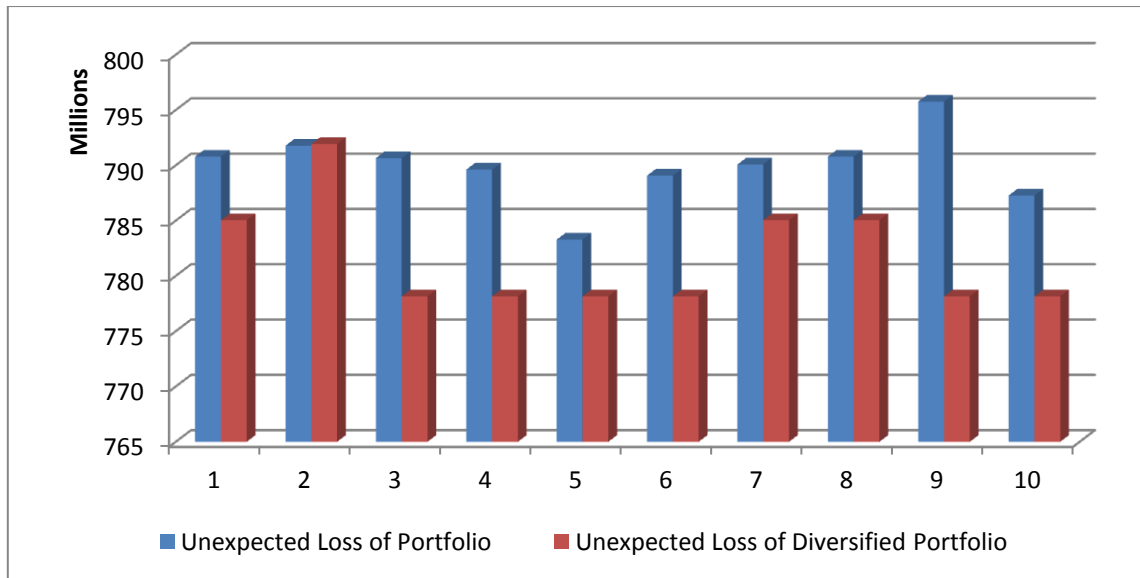


Figure 12.6 Unexpected Losses of Both Portfolio and Equal Exposure Distributed Portfolio

Finally, histogram of portfolio losses demonstrated below:

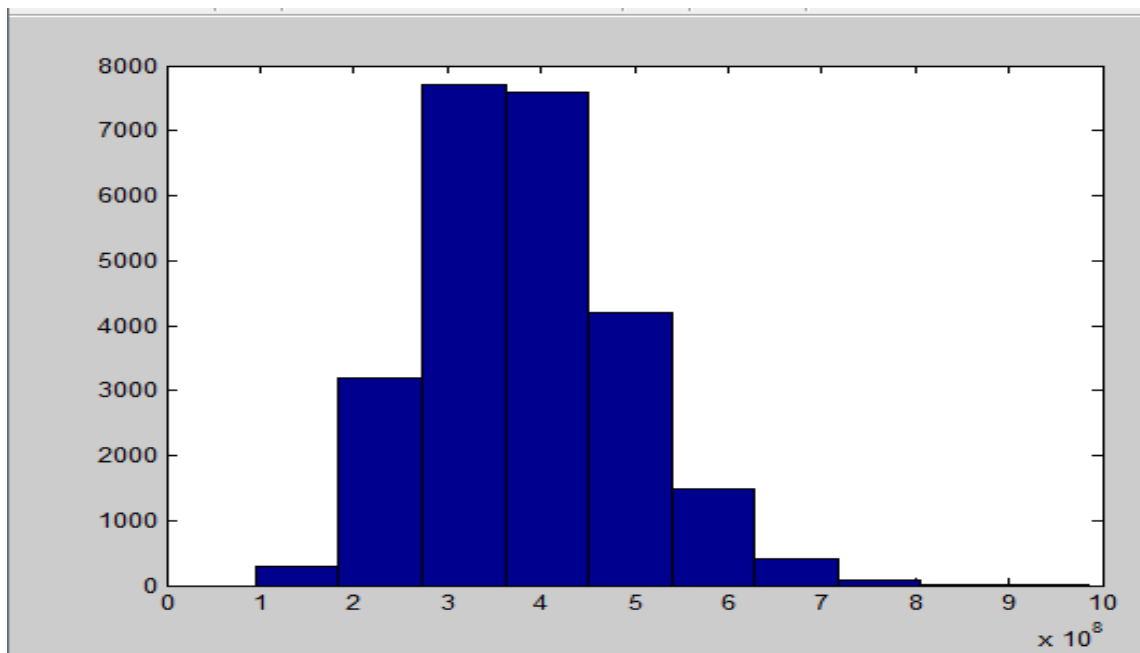


Figure 12.7 Portfolio Loss Distribution from Simulation Results

Portfolio losses histogram above has result in these quantifications below:

Expected Loss: 384455852.96

Unexpected Loss: 790076926.10

Expected Loss of Diversified Portfolio: 450199960.32

Unexpected Loss of Diversified Portfolio: 785039351.95

Credit VaR and credit expected losses of portfolio shown at Appendix. Table result provides capital allocation, risk based pricing and strategic management decisions.

13 Conclusion

Mark to market based models, incorporates the decrease on credit worthiness that results in credit losses. Default models are relative to default probability. Monte Carlo simulation provides determination of rating transitions in an appropriate way for each obligor. CreditMetrics framework based on “Migration Analyses” and default event. Simulation program results mainly shown for default event analysis is more accepted in industries because of its less calibration difficulty

Default correlation risk in credit risk management, is one of the most important risk. In addition to this, the exposure size and concentration risk is unavoidable when you want to manage the portfolio.

General definition of specifying quantile of loss distributions, in a given confidence level alpha, is a value at risk. Quantifying portfolio is the key factor for taking management decisions in institutions. Besides, minimum capital requirement could be measured along with expected loss and unexpected loss of portfolio. Credit VAR provides credit contribution on portfolio unexpected loss and expected loss.

Expected loss of portfolio is lower than diversified portfolio. This is because credit loss is calculated from parameters EAD and LGD when obligor defaults. When portfolio exposure equally diversified, the exposure moved from obligor who has lower PD to higher PD. Thereby portfolio expected loss is getting higher. When it comes to unexpected loss amount, diversified portfolio has lower unexpected loss which is less risky. This is because unexpected loss is equal to portfolio loss at given percentile level. When portfolio is not diversified, the worst scenarios have higher portfolio loss. The economic capital is reduced when portfolio is diversified because of the higher expected loss and lower unexpected loss.

14 References

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Appendix- Global Corporate Annual Default Rates

(%)	AAA	AA	A	BBB	BB	B	CCC/C
1981	0.00	0.00	0.00	0.00	0.00	2.27	0.00
1982	0.00	0.00	0.21	0.34	4.22	3.13	21.43
1983	0.00	0.00	0.00	0.32	1.16	4.55	6.67
1984	0.00	0.00	0.00	0.66	1.14	3.39	25.00
1985	0.00	0.00	0.00	0.00	1.48	6.44	15.38
1986	0.00	0.00	0.18	0.33	1.31	8.33	23.08
1987	0.00	0.00	0.00	0.00	0.38	3.08	12.28
1988	0.00	0.00	0.00	0.00	1.05	3.62	20.37
1989	0.00	0.00	0.00	0.60	0.72	3.37	33.33
1990	0.00	0.00	0.00	0.57	3.57	8.54	31.25
1991	0.00	0.00	0.00	0.55	1.69	13.84	33.87
1992	0.00	0.00	0.00	0.00	0.00	6.99	30.19
1993	0.00	0.00	0.00	0.00	0.70	2.62	13.33
1994	0.00	0.00	0.14	0.00	0.27	3.08	16.67
1995	0.00	0.00	0.00	0.17	0.99	4.58	28.00
1996	0.00	0.00	0.00	0.00	0.67	2.90	4.17

1997	0.00	0.00	0.00	0.25	0.19	3.49	12.00
1998	0.00	0.00	0.00	0.41	0.81	4.61	42.86
1999	0.00	0.17	0.18	0.20	0.95	7.29	33.33
2000	0.00	0.00	0.26	0.37	1.27	7.85	34.12
2001	0.00	0.00	0.27	0.33	3.16	11.25	45.87
2002	0.00	0.00	0.00	1.02	2.85	8.11	44.38
2003	0.00	0.00	0.00	0.23	0.57	4.02	32.93
2004	0.00	0.00	0.08	0.00	0.43	1.56	15.56
2005	0.00	0.00	0.00	0.07	0.31	1.72	9.09
2006	0.00	0.00	0.00	0.00	0.30	0.81	12.38
2007	0.00	0.00	0.00	0.00	0.20	0.25	14.95
2008	0.00	0.38	0.38	0.48	0.79	4.01	26.47
2009	0.00	0.00	0.22	0.54	0.73	10.56	48.94
2010	0.00	0.00	0.00	0.00	0.56	0.82	22.52
2011	0.00	0.00	0.00	0.07	0.00	1.53	15.83
2012	0.00	0.00	0.00	0.00	0.29	1.50	26.62

(Standard & Poors Global Credit Portal, 2013, p: 98- 99)

Appendix- Credits Expected Losses and Unexpected Losses

Exposure	Ratings	Credit Expected loss	Percentage of Expected Loss	Credit Unexpected Loss	Percentage of Unexpected Loss
195,437,820	AAA	2,972,609	1.52%	9,328,497	4.77%
189,000,001	AA	3,461,535	1.83%	10,427,745	5.52%
183,939,953	A	3,956,548	2.15%	11,434,837	6.22%
163,000,000	BBB	4,848,435	2.97%	13,573,518	8.33%
131,000,010	BB	6,231,015	4.76%	17,254,025	13.17%
130,006,270	B	9,337,050	7.18%	23,653,786	18.19%
130,466,887	AAA	1,984,401	1.52%	5,792,106	4.44%
123,506,156	AA	2,262,015	1.83%	6,085,271	4.93%
107,175,124	A	2,305,337	2.15%	6,188,233	5.77%
107,070,328	BBB	3,184,807	2.97%	8,242,909	7.70%
96,918,595	BB	4,609,933	4.76%	11,496,725	11.86%
95,230,172	B	6,839,431	7.18%	15,683,052	16.47%
85,002,380	C	15,790,042	18.58%	33,428,870	39.33%
84,650,220	AAA	1,287,530	1.52%	3,236,715	3.82%
83,473,322	AA	1,528,814	1.83%	3,780,790	4.53%
81,802,260	A	1,759,567	2.15%	4,250,896	5.20%
79,334,817	BBB	2,359,814	2.97%	5,405,283	6.81%
78,179,143	BB	3,718,591	4.76%	8,591,686	10.99%
74,872,863	B	5,377,369	7.18%	12,167,175	16.25%
69,557,041	C	12,920,916	18.58%	26,123,054	37.56%
68,511,815	AA	1,254,794	1.83%	3,028,411	4.42%

55,146,366	A	1,186,198	2.15%	2,702,965	4.90%
55,699,650	AAA	847,192	1.52%	2,119,031	3.80%
57,279,580	AA	1,049,076	1.83%	2,458,420	4.29%
55,244,795	A	1,188,316	2.15%	2,693,612	4.88%
55,676,162	BBB	1,656,087	2.97%	3,836,222	6.89%
55,216,051	B	3,965,617	7.18%	8,703,634	15.76%
50,594,819	C	9,398,494	18.58%	18,569,838	36.70%
48,571,330	AAA	738,770	1.52%	1,838,494	3.79%
47,220,020	AA	864,835	1.83%	2,075,137	4.39%
46,489,018	BB	2,211,250	4.76%	4,757,088	10.23%
43,368,255	B	3,114,708	7.18%	6,452,126	14.88%
42,859,286	AAA	651,890	1.52%	1,553,544	3.62%
42,441,773	AA	777,321	1.83%	1,807,972	4.26%
42,439,339	A	912,870	2.15%	2,074,692	4.89%
41,693,722	BBB	1,240,180	2.97%	2,886,172	6.92%
40,110,208	BB	1,907,842	4.76%	4,034,884	10.06%
40,025,170	B	2,874,608	7.18%	5,826,367	14.56%
39,709,734	C	7,376,480	18.58%	14,263,877	35.92%
38,024,530	B	2,730,922	7.18%	5,759,569	15.15%
37,460,251	C	6,958,616	18.58%	13,386,160	35.73%
37,454,214	AAA	569,679	1.52%	1,374,443	3.67%
34,797,013	AA	637,307	1.83%	1,399,715	4.02%
34,164,506	BB	1,625,035	4.76%	3,403,900	9.96%

33,888,152	B	2,433,847	7.18%	4,791,700	14.14%
32,747,633	C	6,083,200	18.58%	11,525,661	35.20%
32,585,215	AAA	495,621	1.52%	1,112,147	3.41%
32,311,460	AA	591,784	1.83%	1,260,347	3.90%
32,308,739	B	2,320,414	7.18%	4,622,267	14.31%
32,091,701	C	5,961,354	18.58%	11,212,450	34.94%
31,705,451	AAA	482,240	1.52%	1,100,427	3.47%
31,377,230	AA	574,674	1.83%	1,221,894	3.89%
29,728,711	BB	1,414,046	4.76%	2,868,495	9.65%
29,655,926	B	2,129,889	7.18%	4,225,618	14.25%
29,613,043	C	5,500,919	18.58%	10,321,739	34.86%
28,780,005	AAA	437,744	1.52%	937,961	3.26%
28,562,920	AA	523,130	1.83%	1,132,456	3.96%
28,428,438	A	611,496	2.15%	1,274,854	4.48%
28,411,350	B	2,040,503	7.18%	3,999,066	14.08%
27,560,799	C	5,119,694	18.58%	9,740,802	35.34%
27,277,944	AAA	414,898	1.52%	945,008	3.46%
27,273,406	AA	499,512	1.83%	1,074,330	3.94%
26,376,812	AA	483,091	1.83%	1,099,932	4.17%
26,349,978	A	566,788	2.15%	1,323,647	5.02%
25,935,139	BBB	771,441	2.97%	1,747,061	6.74%
25,681,494	BB	1,221,540	4.76%	2,469,745	9.62%
25,522,475	AAA	388,197	1.52%	854,719	3.35%

25,513,587	AA	467,281	1.83%	980,457	3.84%
25,376,962	A	545,858	2.15%	1,168,945	4.61%
25,259,664	AA	462,631	1.83%	931,806	3.69%
25,094,147	BB	1,193,603	4.76%	2,414,870	9.62%
24,792,930	B	1,780,628	7.18%	3,543,832	14.29%
24,685,389	C	4,585,558	18.58%	8,596,268	34.82%
24,345,086	AAA	370,289	1.52%	819,975	3.37%
24,182,629	AA	442,905	1.83%	989,816	4.09%
24,174,623	A	519,996	2.15%	1,104,256	4.57%
23,921,717	BBB	711,551	2.97%	1,408,851	5.89%
23,752,994	BB	1,129,811	4.76%	2,270,569	9.56%
23,651,924	B	1,698,681	7.18%	3,415,640	14.44%
23,252,183	C	4,319,326	18.58%	8,121,046	34.93%
23,155,668	AAA	352,198	1.52%	772,486	3.34%
23,030,231	AA	421,799	1.83%	945,602	4.11%
22,190,187	A	477,311	2.15%	1,010,763	4.56%
21,927,213	BBB	652,225	2.97%	1,320,928	6.02%
21,476,139	BB	1,021,513	4.76%	2,102,524	9.79%
21,423,434	B	1,538,631	7.18%	3,129,553	14.61%
21,095,302	C	3,918,663	18.58%	7,152,550	33.91%
20,677,237	C	3,841,004	18.58%	7,101,006	34.34%
20,066,074	C	3,727,474	18.58%	6,992,819	34.85%
19,862,302	C	3,689,621	18.58%	6,782,912	34.15%

19,493,658	AA	357,026	1.83%	745,366	3.82%
19,342,064	A	416,048	2.15%	917,017	4.74%
19,339,470	BBB	575,253	2.97%	1,251,889	6.47%
19,148,844	BB	910,815	4.76%	1,771,487	9.25%
19,057,234	B	1,368,691	7.18%	2,661,635	13.97%
18,949,726	C	3,520,101	18.58%	6,483,427	34.21%
18,888,061	AAA	287,287	1.52%	648,293	3.43%
18,474,897	AA	338,368	1.83%	677,966	3.67%
18,447,558	A	396,807	2.15%	880,526	4.77%
18,352,185	BBB	545,886	2.97%	1,080,838	5.89%
18,237,733	BB	867,478	4.76%	3,635,317	19.93%
18,221,088	AAA	277,143	1.52%	596,176	3.27%
18,131,679	AA	332,082	1.83%	675,841	3.73%
18,082,756	A	388,960	2.15%	807,428	4.47%
18,065,690	BBB	537,364	2.97%	1,153,208	6.38%
17,965,346	AAA	273,253	1.52%	620,080	3.45%
17,932,399	AAA	272,752	1.52%	624,695	3.48%
17,860,858	AA	327,122	1.83%	718,456	4.02%
17,784,525	A	382,545	2.15%	773,574	4.35%
17,694,774	BBB	526,331	2.97%	1,081,852	6.11%
17,658,071	BB	839,906	4.76%	1,685,685	9.55%
17,500,000	AAA	266,175	1.52%	595,037	3.40%
17,467,558	AA	319,918	1.83%	718,324	4.11%

16,894,469	A	363,400	2.15%	755,453	4.47%
16,875,264	BBB	501,955	2.97%	987,360	5.85%
16,868,123	BB	802,332	4.76%	1,605,945	9.52%
16,800,866	AAA	255,541	1.52%	536,774	3.19%
16,621,881	AA	304,430	1.83%	643,025	3.87%
16,614,230	A	357,372	2.15%	713,077	4.29%
16,611,551	BBB	494,111	2.97%	1,023,085	6.16%
16,402,981	BB	780,208	4.76%	1,546,929	9.43%
16,337,495	AAA	248,493	1.52%	618,397	3.79%
16,316,970	AA	298,845	1.83%	625,995	3.84%
16,270,820	A	349,985	2.15%	736,961	4.53%
16,206,187	BBB	482,053	2.97%	939,895	5.80%
16,198,488	BB	770,481	4.76%	1,453,859	8.98%
16,126,181	B	1,158,182	7.18%	2,265,718	14.05%
16,006,284	AAA	243,456	1.52%	527,817	3.30%
15,925,065	AA	291,668	1.83%	622,198	3.91%
15,904,084	A	342,097	2.15%	663,212	4.17%
15,879,644	BBB	472,340	2.97%	900,582	5.67%
15,848,478	AAA	241,055	1.52%	552,100	3.48%
15,801,980	AAA	240,348	1.52%	525,136	3.32%
15,789,199	AA	289,179	1.83%	576,372	3.65%
15,754,795	A	338,886	2.15%	712,577	4.52%
15,751,977	BBB	468,543	2.97%	967,113	6.14%

15,688,360	BB	746,217	4.76%	1,546,969	9.86%
15,598,577	AAA	237,254	1.52%	544,395	3.49%
15,583,133	AA	285,405	1.83%	583,845	3.75%
15,149,579	A	325,867	2.15%	685,204	4.52%
15,000,604	BBB	446,193	2.97%	920,981	6.14%
14,955,222	BB	711,345	4.76%	1,443,015	9.65%
14,870,775	B	1,068,019	7.18%	2,095,058	14.09%
14,825,475	AAA	225,495	1.52%	497,440	3.36%
14,752,237	AA	270,187	1.83%	582,054	3.95%
14,661,727	BBB	436,113	2.97%	872,897	5.95%
14,654,651	BBB	435,903	2.97%	871,536	5.95%
14,652,399	BB	696,941	4.76%	1,347,994	9.20%
14,640,878	B	1,051,508	7.18%	2,113,391	14.43%
14,586,713	AAA	221,864	1.52%	478,199	3.28%
14,579,520	AA	267,024	1.83%	542,502	3.72%
14,431,790	A	310,428	2.15%	644,406	4.47%
14,423,126	BBB	429,016	2.97%	840,186	5.83%
14,358,941	BB	682,983	4.76%	1,309,942	9.12%
14,330,522	B	1,029,218	7.18%	1,950,911	13.61%
14,324,861	AAA	217,881	1.52%	459,505	3.21%
14,323,668	AA	262,338	1.83%	566,982	3.96%
14,257,668	A	306,682	2.15%	647,608	4.54%
14,244,946	BBB	423,716	2.97%	882,812	6.20%

14,235,690	BB	677,121	4.76%	1,268,560	8.91%
14,202,845	B	1,020,048	7.18%	1,984,556	13.97%
14,171,898	AAA	215,555	1.52%	436,415	3.08%
14,149,270	AA	259,144	1.83%	552,817	3.91%
14,079,558	A	302,851	2.15%	623,259	4.43%
14,030,408	BBB	417,334	2.97%	824,510	5.88%
14,017,598	BB	666,747	4.76%	1,281,500	9.14%
14,007,233	B	1,005,999	7.18%	1,916,785	13.68%
14,001,394	AAA	212,961	1.52%	457,214	3.27%
13,946,389	BBB	414,835	2.97%	857,151	6.15%
13,937,720	A	299,800	2.15%	636,652	4.57%
13,871,309	BBB	412,602	2.97%	816,050	5.88%
13,851,363	BB	658,840	4.76%	1,338,282	9.66%
13,821,219	B	992,640	7.18%	1,878,030	13.59%
13,703,688	AAA	208,433	1.52%	451,009	3.29%
13,686,943	AA	250,676	1.83%	504,021	3.68%
13,678,993	A	294,235	2.15%	620,446	4.54%
13,640,528	BBB	405,737	2.97%	833,977	6.11%
13,599,120	BB	646,842	4.76%	1,290,355	9.49%
13,588,337	B	975,914	7.18%	1,896,076	13.95%
13,578,374	AAA	206,527	1.52%	450,369	3.32%
13,422,763	BBB	399,260	2.97%	820,663	6.11%
13,326,040	A	286,643	2.15%	583,918	4.38%

13,324,999	BBB	396,352	2.97%	808,702	6.07%
13,285,937	BB	631,946	4.76%	1,241,034	9.34%
13,263,975	B	952,619	7.18%	1,821,883	13.74%
13,215,096	AAA	201,002	1.52%	442,559	3.35%
13,185,575	AA	241,494	1.83%	497,401	3.77%
13,173,811	A	283,369	2.15%	581,474	4.41%
13,171,989	BBB	391,801	2.97%	765,614	5.81%
13,152,983	BBB	391,235	2.97%	850,580	6.47%
13,072,677	B	938,880	7.18%	1,829,154	13.99%
13,035,695	AAA	198,273	1.52%	437,387	3.36%
13,031,464	AA	238,671	1.83%	490,751	3.77%
13,004,058	A	279,717	2.15%	568,975	4.38%
12,945,299	BBB	385,058	2.97%	769,048	5.94%
12,882,673	BB	612,764	4.76%	1,257,087	9.76%
12,853,095	BB	611,357	4.76%	1,168,443	9.09%
12,829,318	C	2,383,174	18.58%	4,300,510	33.52%
12,798,191	AAA	194,660	1.52%	422,850	3.30%
12,767,693	AA	233,840	1.83%	520,135	4.07%
12,736,947	A	273,972	2.15%	576,082	4.52%
12,672,237	BB	602,755	4.76%	1,155,254	9.12%
12,634,240	BB	600,948	4.76%	1,249,056	9.89%
12,583,152	B	903,722	7.18%	1,759,852	13.99%
12,571,612	C	2,335,303	18.58%	4,208,479	33.48%

12,568,121	AAA	191,161	1.52%	400,735	3.19%
12,441,453	AA	227,865	1.83%	484,496	3.89%
12,315,235	A	264,901	2.15%	525,405	4.27%
12,248,269	BBB	364,325	2.97%	719,781	5.88%
12,214,871	BBB	363,331	2.97%	725,655	5.94%
12,202,857	B	876,409	7.18%	1,701,967	13.95%
12,192,353	C	2,264,851	18.58%	4,119,064	33.78%
12,167,980	AAA	185,075	1.52%	385,634	3.17%
12,128,803	AA	222,139	1.83%	450,533	3.71%
12,063,087	A	259,477	2.15%	518,517	4.30%
12,054,306	BB	573,363	4.76%	1,145,321	9.50%
11,987,801	BB	570,200	4.76%	1,082,091	9.03%
11,968,895	BB	569,300	4.76%	1,141,046	9.53%
11,948,545	C	2,219,562	18.58%	4,092,654	34.25%
11,926,145	AAA	181,397	1.52%	385,621	3.23%
11,918,051	AA	218,279	1.83%	450,351	3.78%
11,871,592	A	255,358	2.15%	522,472	4.40%
11,837,254	BBB	352,099	2.97%	667,529	5.64%
11,798,818	BB	561,211	4.76%	1,101,366	9.33%
11,793,857	B	847,035	7.18%	1,618,441	13.72%
11,759,364	C	2,184,419	18.58%	4,003,714	34.05%
11,745,382	AAA	178,647	1.52%	364,706	3.11%
11,737,572	AA	214,974	1.83%	457,085	3.89%

11,727,405	A	252,256	2.15%	517,631	4.41%
11,691,584	BBB	347,766	2.97%	717,070	6.13%
11,675,396	BBB	347,285	2.97%	683,119	5.85%
11,663,071	B	837,642	7.18%	1,603,486	13.75%
11,644,090	C	2,163,006	18.58%	3,990,613	34.27%
11,641,202	AAA	177,063	1.52%	384,623	3.30%
11,624,948	AA	212,911	1.83%	433,309	3.73%
11,619,215	A	249,929	2.15%	541,928	4.66%
11,600,177	BB	551,762	4.76%	1,134,918	9.78%
11,567,813	BB	550,223	4.76%	1,011,526	8.74%
11,519,035	B	827,297	7.18%	1,568,165	13.61%
11,494,125	C	2,135,149	18.58%	3,805,009	33.10%
11,371,110	AAA	172,955	1.52%	357,461	3.14%
11,353,790	AA	207,945	1.83%	432,671	3.81%
11,328,875	C	2,104,452	18.58%	3,798,274	33.53%
11,320,649	BBB	336,733	2.97%	666,721	5.89%
11,000,056	C	2,043,370	18.58%	3,684,501	33.50%
11,285,859	AAA	171,658	1.52%	357,678	3.17%
11,000,001	AA	201,465	1.83%	415,660	3.78%
11,269,496	C	2,093,422	18.58%	3,813,793	33.84%
11,000,001	C	2,043,360	18.58%	3,683,072	33.48%
11,264,878	C	2,092,564	18.58%	3,836,802	34.06%
4,086,616	AAA	152,931	3.74%	119,553	2.93%

4,080,388	AA	395,304	9.69%	147,119	3.61%
4,074,513	A	32,361	0.79%	165,728	4.07%
4,068,574	BBB	38,936	0.96%	236,483	5.81%
4,066,089	BB	45,676	1.12%	387,377	9.53%
4,060,544	B	63,113	1.55%	538,983	13.27%
4,058,814	C	100,920	2.49%	1,331,650	32.81%
4,056,738	AAA	151,748	3.74%	124,664	3.07%
4,056,429	AA	391,577	9.65%	147,816	3.64%
4,038,750	A	62,639	1.55%	166,087	4.11%
4,035,953	C	99,920	2.48%	1,353,667	33.54%
4,034,059	C	150,691	3.74%	1,325,081	32.85%
4,032,926	C	389,732	9.66%	1,364,295	33.83%
4,032,236	C	389,712	9.66%	1,357,335	33.66%
4,025,894	C	389,194	9.67%	1,342,803	33.35%
4,023,238	BB	388,632	9.66%	361,355	8.98%
4,014,723	B	388,229	9.67%	527,234	13.13%
4,013,274	AAA	388,068	9.67%	115,347	2.87%
3,989,686	AA	387,432	9.71%	135,658	3.40%
3,989,082	A	85,805	2.15%	182,470	4.57%
3,986,011	BBB	118,564	2.97%	241,146	6.05%
3,977,593	BB	189,194	4.76%	366,187	9.21%
3,965,616	B	284,811	7.18%	539,612	13.61%
3,961,691	C	735,924	18.58%	1,309,189	33.05%

3,960,566	AAA	60,240	1.52%	125,266	3.16%
3,955,842	AA	72,451	1.83%	151,257	3.82%
3,954,926	A	85,070	2.15%	177,102	4.48%
3,953,377	C	734,379	18.58%	1,323,435	33.48%
3,946,015	C	733,012	18.58%	1,325,527	33.59%
3,943,193	C	732,488	18.58%	1,297,764	32.91%
3,942,998	C	732,451	18.58%	1,300,482	32.98%
3,941,754	C	732,220	18.58%	1,283,128	32.55%
3,934,992	AA	72,069	1.83%	150,712	3.83%
3,931,889	A	84,575	2.15%	170,773	4.34%
3,931,263	BBB	116,935	2.97%	219,675	5.59%
3,931,092	BB	186,982	4.76%	363,419	9.24%
3,925,818	B	281,952	7.18%	517,826	13.19%
3,918,308	C	727,865	18.58%	1,308,175	33.39%
3,917,574	AAA	59,586	1.52%	125,163	3.19%
3,914,305	AA	71,690	1.83%	135,104	3.45%
3,912,880	A	84,166	2.15%	179,487	4.59%
3,909,256	BBB	116,281	2.97%	229,731	5.88%
3,903,612	BB	185,675	4.76%	360,377	9.23%
3,902,068	B	280,246	7.18%	530,965	13.61%
3,898,915	C	724,262	18.58%	1,291,445	33.12%
3,884,068	C	721,505	18.58%	1,296,993	33.39%
3,877,281	C	720,244	18.58%	1,291,493	33.31%

3,873,470	C	719,536	18.58%	1,305,382	33.70%
3,867,113	BBB	115,027	2.97%	223,285	5.77%
3,865,494	BB	183,862	4.76%	348,179	9.01%
3,865,305	B	277,606	7.18%	517,035	13.38%
3,861,986	C	717,403	18.58%	1,275,992	33.04%
3,857,478	AAA	58,672	1.52%	127,698	3.31%
3,855,558	AA	70,615	1.83%	129,118	3.35%
3,854,704	A	82,915	2.15%	153,078	3.97%
3,850,406	BBB	114,530	2.97%	221,085	5.74%
3,849,736	C	715,127	18.58%	1,280,342	33.26%
3,844,998	C	714,247	18.58%	1,280,000	33.29%
3,841,611	C	713,618	18.58%	1,256,444	32.71%
3,839,478	C	713,221	18.58%	1,295,405	33.74%
3,838,590	AA	70,304	1.83%	140,863	3.67%
3,831,636	A	82,418	2.15%	155,357	4.05%
3,826,031	BBB	113,805	2.97%	229,995	6.01%
3,824,027	BB	181,890	4.76%	370,449	9.69%
3,822,810	B	274,554	7.18%	530,971	13.89%
3,822,779	C	710,119	18.58%	1,269,660	33.21%
3,813,360	AAA	58,001	1.52%	124,280	3.26%
3,811,515	AA	69,808	1.83%	140,114	3.68%
3,797,760	A	81,690	2.15%	170,308	4.48%
3,797,217	AA	69,546	1.83%	140,807	3.71%

3,797,009	A	81,674	2.15%	160,774	4.23%
3,788,671	B	272,102	7.18%	503,624	13.29%
3,788,357	C	703,725	18.58%	1,260,415	33.27%
3,784,483	AAA	57,562	1.52%	124,310	3.28%
3,778,807	AA	69,209	1.83%	142,548	3.77%
3,775,849	A	81,219	2.15%	163,027	4.32%
3,773,495	BBB	112,243	2.97%	215,459	5.71%
3,757,002	BB	178,702	4.76%	350,217	9.32%
3,755,887	B	269,748	7.18%	515,411	13.72%
3,751,394	C	696,859	18.58%	1,280,126	34.12%
3,731,485	AAA	56,756	1.52%	117,303	3.14%
3,729,877	AA	68,313	1.83%	147,881	3.96%
3,710,929	A	79,822	2.15%	164,986	4.45%
3,707,926	BB	176,367	4.76%	315,907	8.52%
3,699,637	BB	175,973	4.76%	333,714	9.02%
3,697,661	BB	175,879	4.76%	342,076	9.25%
3,693,852	C	686,170	18.58%	1,225,181	33.17%
3,693,071	AAA	56,172	1.52%	110,646	3.00%
3,692,139	AA	67,622	1.83%	138,331	3.75%
3,690,901	A	79,391	2.15%	156,755	4.25%
3,690,213	BBB	109,765	2.97%	198,629	5.38%
3,686,384	AAA	56,070	1.52%	118,959	3.23%
3,685,933	AA	67,508	1.83%	145,903	3.96%

3,685,932	A	79,284	2.15%	156,780	4.25%
3,682,941	BBB	109,549	2.97%	214,305	5.82%
3,680,168	BB	175,047	4.76%	357,457	9.71%
3,679,333	BB	175,007	4.76%	346,282	9.41%
3,678,977	BB	174,991	4.76%	322,646	8.77%
3,675,488	BB	174,825	4.76%	334,837	9.11%
3,675,361	BB	174,819	4.76%	329,402	8.96%
3,660,779	BB	174,125	4.76%	344,300	9.41%
3,659,674	BB	174,072	4.76%	333,866	9.12%
3,655,477	BB	173,873	4.76%	328,324	8.98%
3,652,209	C	678,434	18.58%	1,213,009	33.21%
3,645,043	AAA	55,441	1.52%	120,899	3.32%
3,643,763	AA	66,736	1.83%	136,987	3.76%
3,632,276	A	78,130	2.15%	171,043	4.71%
3,627,190	BBB	107,891	2.97%	223,161	6.15%
3,625,978	BB	172,470	4.76%	330,792	9.12%
3,619,329	B	259,940	7.18%	474,148	13.10%
3,613,517	C	671,247	18.58%	1,218,008	33.71%
3,613,103	AA	66,174	1.83%	133,748	3.70%
3,612,174	A	77,698	2.15%	153,179	4.24%
3,611,110	A	77,675	2.15%	151,049	4.18%
3,610,375	BBB	107,391	2.97%	209,619	5.81%
3,609,271	AA	66,104	1.83%	138,237	3.83%

3,608,137	AA	66,083	1.83%	131,481	3.64%
3,606,141	C	669,877	18.58%	1,184,521	32.85%
3,588,864	AA	65,730	1.83%	124,562	3.47%
3,575,200	A	76,903	2.15%	156,887	4.39%
3,570,942	A	76,811	2.15%	154,409	4.32%
3,570,438	BBB	106,203	2.97%	216,921	6.08%
3,564,668	BB	169,553	4.76%	317,881	8.92%
3,564,053	BBB	106,013	2.97%	219,734	6.17%
3,553,784	BBB	105,707	2.97%	209,981	5.91%
3,552,246	BBB	105,662	2.97%	206,928	5.83%
3,549,170	AA	65,003	1.83%	129,104	3.64%
3,548,907	A	76,337	2.15%	149,358	4.21%
2,465,078	BBB	73,324	2.97%	143,756	5.83%
2,464,381	BB	117,218	4.76%	213,280	8.65%
2,461,764	B	176,804	7.18%	330,872	13.44%
2,456,391	C	456,299	18.58%	830,812	33.82%
2,456,008	AAA	37,356	1.52%	78,940	3.21%
2,449,664	C	455,050	18.58%	809,050	33.03%
2,443,670	C	453,936	18.58%	810,362	33.16%
2,442,107	C	453,646	18.58%	812,194	33.26%
2,441,929	BB	116,150	4.76%	228,570	9.36%
2,435,783	B	174,938	7.18%	337,382	13.85%
2,433,195	C	451,990	18.58%	805,328	33.10%

2,429,885	AAA	36,959	1.52%	80,906	3.33%
2,428,548	AA	44,479	1.83%	91,612	3.77%
2,424,402	A	52,149	2.15%	100,944	4.16%
2,420,639	BBB	72,002	2.97%	143,338	5.92%
2,418,435	BB	115,033	4.76%	226,215	9.35%
2,417,118	B	173,597	7.18%	333,401	13.79%
2,415,337	C	448,673	18.58%	776,019	32.13%
2,413,491	C	448,330	18.58%	794,471	32.92%
2,410,169	AA	44,142	1.83%	91,228	3.79%
2,410,115	A	51,842	2.15%	104,214	4.32%
2,399,508	BBB	71,373	2.97%	135,775	5.66%
2,395,805	BB	113,956	4.76%	220,717	9.21%
2,393,996	A	51,495	2.15%	101,828	4.25%
2,384,855	A	51,298	2.15%	95,166	3.99%
2,382,139	A	51,240	2.15%	101,324	4.25%
2,380,707	AA	43,603	1.83%	86,753	3.64%
2,377,351	A	51,137	2.15%	106,763	4.49%
2,372,692	BBB	70,576	2.97%	143,391	6.04%
2,370,334	BB	112,745	4.76%	214,265	9.04%
2,368,256	C	439,927	18.58%	781,251	32.99%
2,366,762	C	439,650	18.58%	771,193	32.58%
2,365,957	AAA	35,986	1.52%	72,858	3.08%
2,359,204	AA	43,209	1.83%	85,515	3.62%

2,358,742	A	50,737	2.15%	107,138	4.54%
2,358,643	BBB	70,158	2.97%	128,470	5.45%
2,356,598	BB	112,092	4.76%	218,012	9.25%
2,353,859	B	169,054	7.18%	321,957	13.68%
2,353,720	C	437,227	18.58%	780,533	33.16%
2,352,636	AA	43,089	1.83%	83,164	3.53%
2,350,273	A	50,554	2.15%	108,563	4.62%
2,346,525	A	50,474	2.15%	96,647	4.12%
2,325,996	BBB	69,187	2.97%	143,554	6.17%
2,310,862	AAA	35,148	1.52%	77,833	3.37%
2,308,315	AA	42,277	1.83%	91,668	3.97%
2,302,814	A	49,534	2.15%	99,427	4.32%
2,300,810	BBB	68,438	2.97%	130,191	5.66%
2,299,757	BB	109,388	4.76%	212,016	9.22%
2,290,880	B	164,531	7.18%	311,285	13.59%
2,287,812	AAA	34,798	1.52%	75,882	3.32%
2,285,344	AA	41,856	1.83%	87,970	3.85%
2,284,441	A	49,138	2.15%	93,797	4.11%
2,284,156	BBB	67,942	2.97%	143,756	6.29%
2,281,075	BB	108,499	4.76%	210,294	9.22%
2,280,377	B	163,777	7.18%	301,226	13.21%
2,276,274	C	422,841	18.58%	748,279	32.87%
2,276,124	AAA	34,620	1.52%	72,428	3.18%

2,268,546	AA	41,548	1.83%	88,342	3.89%
2,264,574	A	48,711	2.15%	90,657	4.00%
2,261,367	BBB	67,264	2.97%	133,762	5.92%
2,254,303	BB	107,226	4.76%	199,293	8.84%
2,253,448	B	161,843	7.18%	302,006	13.40%
2,252,184	C	418,366	18.58%	733,136	32.55%
2,247,305	A	48,340	2.15%	99,914	4.45%
2,247,127	A	48,336	2.15%	96,013	4.27%
2,246,215	A	48,316	2.15%	97,704	4.35%
2,245,273	BBB	66,786	2.97%	130,505	5.81%
2,245,139	BB	106,790	4.76%	205,685	9.16%
2,238,520	B	160,770	7.18%	295,123	13.18%
2,238,306	C	415,788	18.58%	731,347	32.67%
2,237,933	AAA	34,039	1.52%	69,346	3.10%
2,234,988	AA	40,934	1.83%	81,443	3.64%
2,233,390	A	48,040	2.15%	98,006	4.39%
2,230,626	BBB	66,350	2.97%	124,931	5.60%
2,229,012	BB	106,023	4.76%	210,070	9.42%
2,228,808	B	160,073	7.18%	293,128	13.15%
2,226,946	C	413,677	18.58%	730,635	32.81%
2,216,166	AAA	33,708	1.52%	72,653	3.28%
2,197,853	AA	40,254	1.83%	83,051	3.78%
2,197,397	A	47,266	2.15%	94,734	4.31%

2,192,481	BBB	65,215	2.97%	123,217	5.62%
2,181,338	BB	103,755	4.76%	193,962	8.89%
2,180,790	B	156,624	7.18%	296,186	13.58%
2,180,422	C	405,035	18.58%	722,226	33.12%
2,179,693	A	46,885	2.15%	100,544	4.61%
2,170,682	AA	39,756	1.83%	83,277	3.84%
2,170,242	A	46,682	2.15%	88,412	4.07%
2,169,950	BBB	64,545	2.97%	123,760	5.70%
2,169,808	BB	103,207	4.76%	192,519	8.87%
2,167,152	B	155,645	7.18%	285,853	13.19%
2,165,169	C	402,202	18.58%	715,923	33.07%
2,163,750	AAA	32,911	1.52%	68,852	3.18%
2,163,422	AA	39,623	1.83%	80,084	3.70%
2,161,835	A	46,501	2.15%	91,537	4.23%
2,161,432	BBB	64,292	2.97%	119,947	5.55%
2,155,442	BB	102,524	4.76%	198,988	9.23%
2,149,046	B	154,345	7.18%	286,498	13.33%
2,148,122	C	399,035	18.58%	708,219	32.97%
2,145,615	AAA	32,635	1.52%	71,579	3.34%
2,144,497	AA	39,276	1.83%	82,410	3.84%
2,140,193	A	46,036	2.15%	97,349	4.55%
2,138,780	BBB	63,618	2.97%	129,529	6.06%
2,133,740	BB	101,491	4.76%	198,627	9.31%

2,129,364	B	152,931	7.18%	283,328	13.31%
2,128,035	C	395,304	18.58%	713,747	33.54%
2,127,602	AAA	32,361	1.52%	68,521	3.22%
2,125,919	AA	38,936	1.83%	73,377	3.45%
2,123,493	A	45,676	2.15%	94,000	4.43%
2,121,801	BBB	63,113	2.97%	120,334	5.67%
2,121,734	BB	100,920	4.76%	179,542	8.46%
2,112,899	B	151,748	7.18%	288,863	13.67%
2,107,973	C	391,577	18.58%	705,125	33.45%
2,105,883	BBB	62,639	2.97%	118,620	5.63%
2,100,700	BB	99,920	4.76%	188,139	8.96%
2,098,174	B	150,691	7.18%	278,774	13.29%
2,098,043	C	389,732	18.58%	697,765	33.26%
2,097,933	C	389,712	18.58%	704,862	33.60%
2,095,146	C	389,194	18.58%	703,657	33.59%
2,092,121	C	388,632	18.58%	687,877	32.88%
2,089,949	C	388,229	18.58%	690,247	33.03%
2,089,085	C	388,068	18.58%	686,209	32.85%
2,085,657	C	387,432	18.58%	695,252	33.33%

Tuğba Nalbantoğlu Finansal Ekonomi 2013