İSTANBUL BİLGİ UNIVERSITY
INSTITUTE OF SOCIAL SCIENCES
INTERNATIONAL FINANCE MASTER'S DEGREE PROGRAM

BÜHLMANN STRAUB MODELING WITH CREDIBILITY APPROACHES

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İSTANBUL
2018

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KREDIBILITE YAKLAŞIMLARI ILE BÜHLMANN STRAUB MODELLEMESI

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Tezin Onaylandığı Tarih: 11. 06. 2018

Toplam Sayfa Sayısı: 55.sayfa

Anahtar Kelimeler (Türkçe)	Anahtar Kelimeler (İngilizce)
1) Kredibilite Kuramı	1) Credibility Theory
2) Kredibilite Faktörü	2) Credibility Factor
3) Kredibilite Modelleri	3) Credibility Models
4) Kredibilite Primi	4) Credibility Premium
5) Bühlmann-Straub Modeli	5) Bühlmann – Straub Model

ACKNOWLEDGEMENT

Firstly I would like to extend my sincere gratitute and deep appreciation to my supervisor Assoc. Prof. Genco FAS for valuable help, guidance, support, patience and understanding during thesis work.

I would also like to express my special thanks to the head of our department Prof. Dr. Cenktan ÖZYILDIRIM for continued support, patient and guadiance during my graduation.

I am deeply greateful to my parents, my sisters for their all support and love; special thanks to my nephews Elif Su and Ali Yusuf for their love that motivated me.

Last but not the least; this thesis is dedicated to my mother Zeliha Aydoğdu and my husband Kaan Yavuz who always encouraged and supported me during thesis process. I could not achieve without them.

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ABBREVIATIONS

TSB	Turkey Insurance Association (Türkiye Sigorta Birliği)

SYMBOL LIST

Some of the symbols used in this thesis are presented below along with explanations.

Symbols	*Description*
Z	Credibility factor
μ	Expected value of hypothetical averages
ϑ	Expected value of process variance
α	Variance of hypothetical means
m_i	Risk exposure value during i'th insurance period
m	Total risk exposure value
C	Premium value of credibility

FIGURE LIST

CHART LIST

ABSTRACT

The credibility theory is based on the weighting of the claims data of the previous policy periods and the damage data of the last turn; is the estimated method of calculation used for premium determination for the next policy period.

These calculations are made using the Z credibility factor. Z value can be determined by different process and methods. These processes and methods are called credibility models. The credibility models are examined under two main headings as Limited Fluctuating Credibility and Classical Credibility.

In this study, previous studies on credibility theory and credibility theory have been examined. The connection between the Bayesian Approach, Bühlmann and Bühlmann-Straub credibility models from the forerunner of credibility theory is theoretically explained. By choosing the Bühlmann - Straub method from these models; Turkey Insurance Association received from the 2014, 2015, 2016 and 2017 the amount of damage damage insurance with traffic data relating to were made modeling. There are two calculations for damage modeling operations. The first calculation is the credibility account taken from the past years' averages. The second calculation is the calculation of credibility with independent randomly selected data sets.

Finally, the damage model is interpreted according to whether the credibility factor is 0, 1 or within this range.

Key Words: Credibility Theory, Credibility Factor, Credibility Models, Credibility Primer, Bühlmann Model, Bühlmann - Straub Model,

ÖZET

Kredibilite kuramı önceki poliçe dönemlerine ait hasar verileri ile son döneme ilişkin hasar verilerinin ağırlıklandırılması ile birlikte; bir sonraki poliçe dönemine ait prim belirleme işlemi için kullanılan tahmini hesaplama yöntemidir.

Bu hesaplama işlemleri Z kredibilite faktörü kullanılarak yapılmaktadır. Z değeri ise farklı yöntem ve metotlar ile belirlenebilmektedir. Bu yöntem ve metotlar kredibilite modelleri olarak adlandırılmaktadır. Kredibilite modelleri Sınırlı Dalgalanmalı Kredibilite ve Klasik Kredibilite olmak üzere iki ana başlık altında incelenmektedir.

Bu çalışmada kredibilite kuramı ve kredibilite kuramı üzerinde daha önce yapılan çalışmalar incelenmiştir. Kredibilite kuramının öncülerinden Bayesci Yaklaşım, Bühlmann ve Bühlmann - Straub kredibilite modelleri arasındaki ilişki kuramsal olarak açıklanmştır. Bu modellemelerden Bühlmann - Straub yöntemi seçilerek; Türkiye Sigorta Birliğinden alınan 2014, 2015, 2016 ve 2017 yıllarına ilişkin trafik sigortası hasar verileri ile hasar tutarı modellemesi yapılmıştır. Hasar tutarı modelleme işlemleri için iki şekilde hesaplama yapılmıştır. İlk hesaplama geçmiş yılların ortalamaları alınarak yapılan kredibilite hesabıdır. İkinci hesaplama ise birbirinden bağımsız rastgele seçilen veri kümesi ile yapılan kredibilite hesaplamasıdır.

Son olarak da hasar tutarı modellemesi kredibilite faktörünün 0, 1 veya bu aralıkta olması durumlarına göre yorumlanmıştır.

Anahtar Kelimeler: Kredibilite Kuramı, Kredibilite Faktörü, Kredibilite Modelleri, Kredibilite Primi, Bayesci Yaklaşım Bühlmann Modeli, Bühlmann – Straub Modeli,

INTRODUCTION

In actuarial science, the word "credibility" was first used as a measure of confidence in the extent to which actuaries should rely on the experience of a number of insurers when making pricing. The credibility word is defined as a measure of the experience of a number of insurers for actuarial premium determination. Briefly, credibility is called the measure of trust in actuarial science. (Longley-Cook 1962)

According to Frees et al. (2005), credibility theory examines the methods used to predict the future damage of any unit in a group of similar risk units with known historical damage information.

When credibility theory is examined in detail, the main aim is to calculate the fair premium values and interpret the calculated results using the policyholders' experiences and past experience with specific weighting methods. This weighting is based on the credibility factor. The credibility factor will give a weight to the past insurance data, a specific weight to the current data.

At this point, homogeneous or heterogeneous insurance portfolio is important for premium. If the insurance portfolio is homogeneous, using the overall average is the right approach. When examining the insurance sector data, it is seen that the insurance portfolios are heterogeneously distributed. In this case it would be unfair to use a common premium. Because some insurers have good damage experiences, when there is no damage or the number of damage is small, some insurers have bad damage experiences, where the number of damage are big. In this context, it is important for insurance companies to distinguish between good and bad insurance holders. Determining fair premium values for both insured and insurance companies will help to ensure that sector fairness.

The credibility theory also allows the calculation of the premium values that will provide compromise within the two parties by giving specific weight to the two premium values, that is to say the premium values determined both in terms of insured and insurance companies; all of the transactions and methods are examined under credibility theory.

The purpose of studying credibility in this thesis is; is to introduce the Bühlmann - Straub credibility model as a source of work to be done using classical credibility models. In this direction, the history of the Bühlmann-Straub credibility model was searched; its theoretical structure has been examined and summarized. Then, Bühlmann-Straub credibility models functioning describes using data taken from the Turkey Insurance Association (TSB) website. Compensation payments related to the vehicle type, the number of damages related to the vehicle type, the premium value received and the number of insurance policies (number of insured persons); Bühlmann-Straub credibility model and the model of the damage value model. For these modeling operations, damage data between 2014 and 2017 are used and 16 quarters of insurance data are included in the calculation. Using these data and the Bühlmann-Straub credibility model, premium values were obtained for the next insurance period. After the obtained values, the results were compared and the results of the comparisons were interpreted.

CHAPTER I

LITERATURE REVIEW AND METODOLOGY

1.1 THESIS CONTRIBUTION

1.1.1 Literature Review

When the literature survey was carried out, it was seen that the persons who studied the Bühlmann credibility method had the following points.

Meral Ebegil published her article in 2006, “A Study on Bayesian Method in Credibility”; credibility is mentioned in the following scope;

Bühlmann and Bühlmann-Straub credibility models are the best linear approximations to the Bayesian standard. Hence, in the study relationship between Bayesian method and Bühlmann credibility model are shown theoratically.

The relationship between the credibility theory and the Bühlmann-Straub model and the one-factor random effects model, which is a special case of the mixed effect model, was theoretically examined in the Meral Ebegil’s doctoral dissertation titled "Parameter Estimation Credibility Theory and A Statistical Approach" in 2007. The Bühlmann-Straub credibility model and the single-factor random effects model were used to calculate the damage estimates for both models.

Aslıhan Şenturk and Cenap Erdemir published an article in 2010, " Panel data models in credibility theory and an application for the motor third party liability insurance"; credibility theory and panel data regression models are explained in general terms, the link between the credibility models and panel data models is investigated. Bühlmann, Bühlmann-Straub and Jewell’s hierarchical credibility models that are given under greatest accuracy credibility approach, are examined as special cases of mixed linear models.

Abdurrahman Erdal made a study on credibility models in his master thesis titled "Studiying Credibility Factor in Classical Credibility Models" published in 2013 and modeled the amount of damage with the help of Bühlmann-Straub model. In addition, Monte Carlo Simulation method on Matlab has examined the interval estimates of the credibility factor.

After 2013, studies on credibility and credibility motives were carried out. However, after 2013, there was no study in the literature on Bühlmann-Straub Credibility Model.

1.1.2 Research Questions

This thesis aims to be able to make credibility calculations of traffic and insurance files by using real data after examining the credibility models and to be a reference for the future work to be done on this field.

The specific research questions are reported below:

1) What is the credibility meaning?

2) What is credibility history?

3) How many credibility models are there?

4) What are the similarities and differences between credibility models?

5) Is there a link between the techniques and methods used for credibility modeling? If so, how is it evaluated?

6) What are the benefits of credibility calculations for the insured and insurance company?

7) What are the difficulties in calculating the real data in the insurance sector and the credibility of motor vehicles and traffic files?

According to the research questions we tried to help advance research by providing following contributions:

1) The importance of Z credibility in the premium calculation to be modeled for traffic insurance
2) Bühlmann-Straub's calculations by taking the average of the past years and emphasizing the importance of the used data by revealing the differences between the independent data

1.1.3 Methodology

This section describes the methodological approach that explains how research work is done to find answers to the proposed research questions.

The selection of the research field and contextual information about credibility modeling will be evaluated. First of all, the use of credibility theory and how credibility modeling should be done will be investigated. Here, credibility modeling operations history will be investigated first; scientific and technical resources will be used for these transactions.

The difficulties that may arise in the modeling process of credibility theory, the feasibility of the models, and the differences between the models will be examined. In addition, similarities and differences between credibility models are explained.

The Bühlmann-Straub credibility model was chosen among the models examined in this study; credibility account will be made with traffic insurance data obtained from the Turkey Insurance Association. The data to be used before the credibility account was prepared and the past details such as compensation amounts, number of insured persons, number of claims, premiums received, number of files, etc. The data belongs to 16 cycles as 4 years.

Values such as mean μ, variance ϑ were calculated with these data. The credibility factor Z was then calculated. Two different methods have been used to find the credibility premium. In the first of these methods, the average value of the past years is used instead of $\bar{X}$ in the credibility premium calculation, and randomly selected values are used in the second method.

1.1.4 Structure of Thesis

The reminder of this thesis is structured as follows. The thesis has two parts, theoretical and practical. The theoretical part consists of the following sections:

In the first part of the thesis, literature review, research questions and thesis contribution, methodology used in thesis are mentioned.

In second part of thesis, credibility theory is discussed and concepts that constitute credibility are mentioned.

In the third part of thesis, studies carried out in the area of credibility to date have been examined historically. From Mowbray, who first used the concept of credibility, Bailey and Bayesen to Bühlmann assessed the studies that deal with the different points and similarities of the credibility developed and progressed in history.

In the fourth part of thesis; The main and sub-headings of the credibility models laid out in the studies conducted in relation to the historical behavior of the credibility are briefly explained. Under the models, the concepts of credibility are mentioned.

The practical part consists of the following sections.

In fifth part of thesis, examples of traffic files and credibility calculations taken from the insurance sector will be detailed. Here credibility calculations will be done by calculations made with sample; the size and content of the data sets and objects will be determined. The scenarios modeled at this stage will be supported by examples, outputs and outputs. The credibility factor, which has an important place in the credibility models, was examined and the damage amount modelling was done on the traffic insurance data using the Bühlmann-Straub credibility model.

The last part of thesis, the resulting credibility calculator analytical calculations will be presented in a report and the advantages and disadvantages will be assessed.

CHAPTER II
CREDIBILITY THEORY

2.1 CONCEPTS OF CREDIBILITY

2.1.1 Credibility Definitions

To explain the credibility theory, first explain the concepts related to insurance. These concepts will focus on the concepts defined by Klugman et al. (1998).

Definitions of some important concepts used in insurance and credibility calculations are as follows.

Risk: It is defined as an uncertainty that may result in loss or an insured hazard.

Personal risks: Personal risks: Directly affect the individual. The risk of loss of income upon loss of ability to earn income is the risk.

Specific risks: Requires losses due to individual events and affects individuals rather than the entire group. Sometimes the individual is caused by his own fault.

Fundamental risks: risks that affect a group in the economy or a large group of people, or the entire economy.

Pure risks: The risk of damage is present or the situation continues in the same way.

Uncertainty: The uncertainty arises from the lack of information about what can not happen in the future.

Insurance: A large number of individuals facing the same risk to bring the probability of damage, which is uncertain as risk management and risk management in order to meet the by a person or organization with responsibility.

Insurance policy: A contract that says that an insurer who receives a certain amount of premium, called a premium, agrees to pay a certain amount of what is called an insured amount when a particular damage event occurs, or to a third party who is liable if the insurer is responsible.

Insurer: An insurer is a person who is a party to the insurance contract and who carries out risk management by bringing together the persons who face the same risks, not the damage itself, and collecting the premiums received from them in a pool to meet the individual risks, and institutions.

Premium (premium): The amount paid by the policyholder to the insurer in exchange for insurance coverage.

Insured: It is one of the parties to the insurance contract. By paying the premium, he is the person who takes over the debt and liabilities arising out of the agreement, the transfer to the insurance company and the contrary.

Loss event or claim: The insured or a group of insured persons must be exposed to potential collateral damage by insurance contracts.

Loss: The cost of a loss that a group of insurers or a group of insurers are exposed to as a result of a loss. The loss resulting from damage can also be zero.

Credibility: The process of combining past experiences of this group or individual to better estimate the amount of loss expected of a group or individual (or to estimate any statistical magnitude related to them) is called credibility.

Credibility factor: When the formula C = ZR + (1-Z) H is used, the value of Z is either credibility or credibility factor. The credibility factor Z, expressed by below formula:

$$Z = \frac{n}{n + k}$$

Where n is the number of trial periods or risk exposures and k is the number of units exposed,

$$k = \frac{expected\ value\ of\ process\ variance}{variance\ of\ hypothetical\ means}$$

Riske Exposure Unit: The basic unit of measure for the determination of premiums (Klugman et al. 1998).

2.1.2 Credibility Theory

The credibility theory emerged in the late 1890s with the pricing studies of employers' liability insurance. [Mowbray, 1914].

Mahler et al. 2001, Credibility theory provides various means to deal with the coincidence of the data used to predict future events or costs. For example, an insurance company uses past insurance claims and other information to estimate insurance premiums. Because insurance losses are caused by accidental events, the premium estimate must be made reasonably. The expected correctness of this estimate is a function of the variability in losses.

Credibility theory is interested in estimating the expected claims for the next policy period by using the prior claim data. In a crebibility model, the observed claim data for each risk are used as prior data and the density of the risk parameter is called the structure function. The easiest way of the calculating credibility premiums is the linear Bühlmann credibility method. (Ebegil 2007)

Statistically, credibility theory is a method that aims to achieve in a non-intuitive way. In the presence of preliminary information, sample averages or other unbiased estimators can be used. However, credibility theory gives this experiment only a partial weight and the remaining weight gives the estimator of the other (current) information. [Klugman ve ark., 2004].

C: Credibility estimator (number of damage, amount of damage, risk premium, etc.)

Z: Credibility factor (weight given to current observations)

$\bar{X}$: Average of current observations

μ: Preliminary average

$$C = Z\bar{X} + (1 - Z)\mu \quad (2.1)$$

Calculated from the above equation [Herzog, 1994].

Credits calculated by weighting the credibility factor in equation (2.1) are called credibility premium [Kaas ve ark., 2001].

CHAPTER 3

HISTORY OF CREDIBILITY

3.1 CREDIBILITY RESEARCHES

The credibility theory emerged in the late 1890s with pricing studies of employers' liability insurance. The first definition of credibility is given for full credibility in the work undertaken by Mowbray. This study, which is the beginning of credibility theory, sets the criteria for giving full credibility to data (Mowbray, 1914; Ebegil, 2007).

When Mowbray (1914) 's study is examined, credibility is defined as the work done against the risk of policyholder' s knowledge and experience on policy pricing. For this reason, credibility is one of the most important methods of casualty actuarial science Mowbray (1914) summarized his work on credibility with the text "How many trials / results need to be observed before I can believe my data?". He pointed out that the number of data is important for achieving correct credibility.

In addition, Mowbray (1914) conducted a study on the minimum number of observations required for full reliability in work accident insurance. In these studies, the probability of change determined for the number of observations was assumed to be binomial distributed, and the lower limit of the number of observations was found for normal convergence and full confidence. (Mowbray 1914)

Following Mowbray, Whitney (1918) used existing estimates and new data to obtain new estimates; it is focused on combining these data.

Whitney (1918), brought the concept of Partial Credibility into the agenda, thinking that it would be difficult to achieve complete credibility, and made experience pricing by weighting individual risk with all risk class data. (Whitney 1918)

According to Goulet (1918) Whitney's partial credibility approach constitutes the first step of the Largest Accuracy Credibility because it is based on homogeneity. [Goulet 1998].

Arthur Bailey (1945 and 1950) published his least squares approach to classical credibility and found that the equality of the credibility estimator would be derived from Bayesian theorem. For this reason, Arthur Bailey is known as the pioneer of modern credibility theory (Bailey 1945, Bailey 1950).

Basically Bailey's work is about associating with Largest Accuracy credibility and Bayes Theorem. This has brought a different perspective to credibility theory. A Bayesian Approach has been adopted since it is not right to estimate the credibility premium according to the information of the learner without considering the preliminary distribution (Bailey 1945)

Bühlmann constructed the credibility model of Bühlmann by obtaining a credibility formula, which can be expressed as the best linear approach of Bayes' method independent of the distribution (Bühlmann, 1967). Following this work, Bühlmann and Straub (1970) presented the Bühlmann-Straub model of credibility, which is a generalized version of the Bühlmann model, taking into account the fact that the unit numbers exposed to risk vary according to the individual. In the Bühlmann-Straub credibility model, the variance homogeneity assumption is impaired and the feature of variance heterogeneity is added (Bühlmann and Straub, 1970).

The credibility theory has entered a rapid development process since the 1970s. Jewell (1975) worked with a two-step hierarchical model and extended Taylor's (1979) model with a more gradual model (Kremer, 1994, Ebegil, 2007).

Work on credibility theory has focused on generalizations between 1970-1980 and on predictors of parameters since 1980. Important studies of the period have been carried out by Norberg, Gisler, De Vylder, Dubey and Gisler (De Vylder, 1981, Kremer, 1994, Schnieper, 1995). The process of decelerating from the mid-1980s to the 1990s was accelerated in the 1990s and the work concentrated on optimal parameter estimators.

Campbell used the credibility theory to classify car models (Campbell 1986).Sundt (1987), is another work used in the classification of vehicles by the Theory of Credibility.

According to Dannenburg; Some statistical models, well known in recent years, are applied to credibility theory. As an example of these studies, models of variance components can be given (Dannenburg, 1995).

In the following years, some well-known statistical models have been applied to credibility theory. As an example of these studies, models of variance components can be given [Dannenburg, 1995]. Dannenburg et al. (1996) stated that analysis would be difficult because of the multiple risk parameters between groups or individuals when the risk parameter condition, which is difficult to explain the credibility models, is formed, and it would be easier to divide the risk variables into independent variance components instead of using the variance components model in obtaining the credibility estimators [Dannenburg et al. 1996].

Dannenburg et al. (1996) stated that analysis would be difficult because of the multiple risk parameters between groups or individuals when credential models are difficult to explain the risk parameter condition, and it would be easier to divide the risk variables into independent variance components and that the variance components (Dannenburg et al., 1996). Young developed a credibility formula that minimized the damage function using the decision theory in 1997 (Young, 1997).

Goulet (1998); In his study of all credibility models, such as Limited Fluctuating Credibility, Largest Accuracy Credibility, Bayesian Credibility, Bühlmann-Straub Credibility, Hierarchical Credibility and Cross-Classification Credibility Models, he made parameter estimates and explained in which situations the models would be used (Goulet 1998).

Norberg (2004), argues that the Limiting Fluctuating Credibility approach can be called the "Stable Effect" theory of credibility and the "Largest Accuracy Credibility" approach can be called the "Random Effect" theory of credibility (Norberg 2004).

Mahler et al. emphasized fundamental assumptions and model types of the Theory of Credibility (Mahler et al. 2005).

Ebegil (2006) has theoretically established the relationship between the Bayesian method and the Bühlmann credibility model.

Bühlmann-Straub credibility model is described a generalized version of the Bühlmann credibility model, as a special case of the mixed effect model, using the best linear unbiased estimators to "predict" the creditworthiness premium prices. (Ebegil 2007)

Kaas et al. (2008), have addressed credibility problems as bold guessing problems and have focused on the Largest Accuracy Credibility approach.

Gau et al. (2008) examined the range estimation of the credibility factor, especially the credibility factor.

Wen and Deng (2009) examine credibility premiums when the risks are dependent. Bühlmann developed a credibility formula to explain equal correlations of risks and took into account parameter estimates in this model [Wen and Deng, 2009].

Atanasiu (2009) presented some extensions of the Bühlmann-Straub credibility model. He then defined the updated credibility formula based on covariance structures. It has been shown that the newly adjusted credit premium can be calculated as the weighted average of the net premium price in the previous period and the current period loss [Atanasiu V., 2009].

Şentürk basically explained the credibility theory and mentioned his approaches in his thesis study prepared in 2010. The application part of the study group data Şentürk prepared the panel considering individual data such as data analysis and credibility premium account is made for all the cities in Turkey (Şentürk 2010).

In the presence of preliminary information, sample averages or other unbiased estimators can be used. However, credibility theory gives this experiment only partial weight and the remaining weight gives the estimator of other (current) information. The credibility theory includes methods that enable the insurer to quantitatively solve this problem. (Klugman et al., 2004, Ebegil, 2007, Erdal, 2013).

In Karadağ (2014) thesis research, the Generalized Linear Models (GDM) and Creditworthiness Theory, which are frequently used in the pricing of non-life insurance products, are discussed together.

It has been investigated how linear model components generalized by Erdemir and Sucu (2016) affect the model's credibility. The structure of the connective tissue and the effect of retribution were investigated.

CHAPTER IV

CREDIBILITY APPROACHES

4.1 CREDIBILITY MODELS

The credibility theory examines the methods used to predict the future damage or amount of damage to a future unit in a group of similar risk units with known historical damage information (Frees and Wang, 2005).

Credibility theory; is a method for calculating the weighted estimated value of the previous period insurance data and the new term insurance data; the weighting process of the data is done by Z value. Z value is the credibility factor and credibility models are used to find the Z value.

The credibility models are grouped under two main headings. These main groups are Limited Fluctuation Credibility and Classical Credibility. Limited Fluctuation Credibility Limited fluctuating credibility consists of two subtitles. These subheadings are; Full Credibility and Partial Credibility.

Classical Credibility consists of three subtitles. These are the Bayesian Approach, the Bühlman Credibility Model and the Bühlman Straub Credibility Model.

- Limited Fluctuation Credibility
  - Full Credibility Approach
  - Partial Credibility Approach
- Classical Credibility
  - Bayesian Approach
  - Bühlmann Credibility Model
  - Bühlmann Straub Credibility model

4.1.1 Credibility Approaches

The credibility theory is grouped according to the importance of different measures in previous studies. This grouping is modeled by different perspectives from different perspectives.

According to Goulet (1998), credibility has been examined under two main headings: Limited Fluctuating Credibility (American Credibility) and Largest Accuracy Credibility (European Credibility). Largest Accuracy of Credibility by Goulet; are seperated Bayesian credibility and Bühlmann-Straub credibility. Under the Bühlmann-Straub credibility model, the hierarchical credibility model and the cross-classification credibility model have been examined as sub-titles.

According to Dannenburg et al. 1996, credibility has been examined under 5 main headings. These; Limited Fluctuating Credibility, Largest Accuracy Credibility, Bühlmann-Straub Model, Jewell "In Hierarchical Model and Hachemeister Model.

According to Klugman et al. [1998], Mahler and Dean [2005] and Herzog [1999], credibility approaches is divided into three section as Limited Fluctuation Credibility (Full and Partial), Bayesian Credibility and Largest Credibility of Accuracy (Bühlman and Bühlmann-Straub).

According to Kaas [2008], the credibility approach is divided into two parts: Limited Fluctuating Credibility and Largest Accuracy Credibility (Bühlmann, Bühlmann-Straub and Bayesian).

Key Assumptions About Credibility Theory:

X_{ij}, (i = 1, 2, ..., r; j = 1, 2, ..., n) j'th year, i. the average of the amounts of the losses related to the risk, i. the average damage amount for the risk can be defined by the following formula.

$$\overline{X_\iota} = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \tag{4.1}$$

i) For each i risky, there exists a parameter Θi $(i = 1,2, ..., r)$ and Θ_i are independent and coincident random variables.

ii) $[(X_{i1}, X_{i2}, ..., X_{in}, \Theta_i): 1 \leq i \leq r]$ is independent and has the same distribution.

iii) For any i risk, X_i is the conditional independent variable of $X_{i1}, X_{i2}, ..., X_{in,}$ where i is the random variable.

In this direction, Θ_i, i'th risky is the parameter of damage distribution.

$$m(v)=E[X_{ij}|\ \Theta_i=v] \rightarrow \Theta_i=v \quad (4.2)$$

is the conditional expected value of the total damage.

The variance is defined under $\Theta_i = v$ condition as

$$s^2(v)=Var[X_{ij}|\Theta_i=v] \quad (4.3)$$

and, the overall mean of the total damage is $\mu = E\ [m\ (\Theta_i)]$. If Θ_i is known, $m\ (\Theta_i)$ can be used for premium calculations. Since the risk parameter is usually an unknown parameter, estimators are needed. $\bar{X}_i$ can be a predictor of $m\ (\Theta_i)$.

4.1.2 Limited Fluctuating Credibility

Limited fluctuating credibility is the first work done by Mowbray (1914) and Whitney (1918) to quantify the concept of credibility. In the limited fluctuating credibility theory, frequentist models are used to determine the credibility factor Z and to find the expected number of claims required for full credibility. Many methods of using statistical methods in credibility theory have also been studied by Bailey [Bailey, 1950]. These include Bayesian analysis, least squares approach and regression analysis. In a limited fluctuating credibility theory, Goulet conducted a study [Goulet, 1997].

Theoretically mention of limited fluctuating credits;

If a fuse $i \in \{1, 2, L, n\}$ you have to insure Y insured persons during the i'th experience period. In other words, i; Y is the past experience of belonging to the i'th insurance or i'th insured in a particular group for pricing.

$E(Y_i) = \mu$ In this case, the average is the same as the time or class members.

Assume for all i; $Var(Y_i) = \sigma$

By going out of the data; $\bar{Y} = (Y_1 + Y_2 + L + Y_n)/n$ is the average of past experiences.

If $E(\bar{Y}) = \mu$ and Y_i 's are indepedent; the formula is $Var(\bar{Y}) = \sigma^2/n$.

These ways Y_1, Y_2, L, Y_n 's are independent and have same distribution. (Ebegil, 2007)

4.1.2.1 Full Credibility

The consequences of the insured's damage experience are deducted from future insurance premiums according to their own past experience. At this point, the number of observations is important. It is important that there is sufficient insurance coverage to reveal full credibility, ie, $Z = 1$. In a full credibility approach, "experience" is measured specifically with the help of the expected number of damages over the next period of observations.

In short, insureds with sufficient number of claims usually require that their future premiums be based on their own knowledge. The important situation here is that when the insured experience (how many years, how many years later) is large enough to provide full credibility. (Ebegil, 2007)

4.1.2.2 Partial Credibility

There is a need for a new estimator to be used in the premium calculations. The credibility estimator that can be used until full credibility is reached is defined as follows:

C: Credibility estimator (number of damage, amount of damage, risk premium, etc.)

Z: Credibility factor (weight given to current observations)

$\bar{X}$: Average of current observations μ: Preliminary average

$$\hat{C}_i = Z\bar{X}_i + (1 - Z)\mu \qquad (4.4)$$

$Z \in [0,1]$ and Z is called the credibility factor. When the above equation reaches full credibility, $Z = 1$ and $\bar{X}_i$ can be used in premium calculations. The information that $Z\bar{X}_i$ shows that until it reaches full credibility is insecure or insufficient. The Z credibility factor is a factor that increases with time and approaches to 1. (Klugman, 1992)

Chart 4.1 Differences Between Full and Partial Credibility – (Karadağ 2014)

Full Credibility	**Partial Credibility**
It deals with the number of observations required for complete trust.	Z is dealt with when there are fewer observations than n for the number of observations required for complete confidence.
There are n or more data.	There are fewer than n observation.
Z=1	$0 \leq Z \leq 1$

4.1.3 Classical Credibility

Classical credibility is consists of three sections.

4.1.3.1 Bayesian Approach

In the stack, the distribution of risk characteristics is represented $\pi(\theta)$. In addition, given the θ value of the experience of any insurance with a risk parameter θ, $f_{Y|\Theta}(y \mid \theta)$ be the conditional distribution of the losses or damages. To be charged for Y_{n+1} when $Y = y$ is observed for any insured. For the insured, the risk parameter θ is to be independent of each other, corresponding to the different risk exposure periods of the insured. Statistically, the damage or losses under θ condition, $Y_1, Y_2, \ldots, Y_n, Y_{n+1}$, are independent of each other.

Y, conditional probability function;

$$f_{Y|\Theta}(y_t \mid \theta) \quad ; t=1, 2, \ldots, n, n+1 \tag{4.5}$$

If θ value is known, hypothetical mean;

$$\mu_{n+1}(\theta) = E[Y_{n+1}|\Theta=\theta] = \int y_{n+1} f_{Y_{n+1}|\Theta}(y_{n+1}|\ \theta) dy_{n+1} \tag{4.6}$$

Also the expected value of hypothetical mean; is written as below.

$$\mu = E[Y_{n+1}] = E[E(Y_{n+1}|\Theta=\theta)] = E[\mu_{n+1}(\theta)] \tag{4.7}$$

Therefore, the common premium is the average of the hypothetical average over Θ. If no information is available about the insured person, this premium can be used. However, since the θ value is not known, the best option is to use the observed data. Therefore, Bayesian premium

$$E[Y_{n+1}|Y=y] = \int y_{n+1} f_{Y_{n+1}|Y}(y_{n+1}|\ y) dy_{n+1} \tag{4.8}$$

Or;

$$E[Y_{n+1}|Y=y] = \int \mu_{n+1}(\theta) . \pi_{\Theta/Y}(\theta|y)\, d(\theta) \tag{4.9}$$

As a result, the Bayesian premium can be defined as the expected value of the hypothetical average over the expected value of the estimator distribution or the posterior distribution $\pi_{\Theta/Y}(\theta|y)$. (Klugman et al. 1998)

4.1.3.2 Bühlmann Credibility

In this credibility model, which is also called the Least Squares Credibility, it is to minimize the error between the previously estimated amount and the actual value.

Bühlmann uses the variance of expected values and hypothetical averages of the process variance.

In this credibility approach, it is important that experiences are homogenous; in a heterogeneous portfolio individual experiences become even more important. Bühlmann credits the observed data and priori knowledge. Since the observed data alone is not reliable, more secure data is obtained using priori information. The role of preliminary averaging is the measure of information about all data Mahler (2005). Bühlmann credibility factor is found below equation.

$$\hat{\mu}_{BühlmannCredibility\ (i)} = Z\bar{X}_i + (1 - Z)\mu \tag{4.10}$$

In the Bühlmann credibility approach, the credibility factor is found by the ratio of the expected value of process variances to the variance of hypothetical averages.

$$Z = \frac{n}{n + k_{Bühlmann}}$$

$$k_{Bühlmann} = \frac{Expected\ Value\ of\ Process\ Variance}{Variance\ of\ Hypothetical\ Means}$$

calculated from the formula above Herzog (1999). $k_{\text{Credibility}}$ parameter and model parameters. As the number of observations increases, the Z value approaches 1, but never takes a value of 1. $Z = 1$ means only for full credibility. Since Bühlmann credibility is related to the priori distribution, it is handled in some sources together with Bayesian credibility.

4.1.3.3 Bühlmann Straub Credibility Model

Bühlmann-Straub Credibility is a generalization of Bühlmann credibility. The Bühlmann-Straub model differs from the Bühlmann model in that the weight factors are used because the variances are heterogeneous.

The Bühlmann model requires that a fuse's past damage results (values) consist of components that are independent of each other and have the same distribution for each previous period. One of the difficulties that this hypothesis presents is that it does not allow changes in the value of exposure. In other words, the Bühlmann model does not allow for different numbers of units exposed to risk in past policy periods or distributions of different sizes. That is, the Bühlmann model is not a structure to deal with such differences. One of the most important constraints for the Bühlmann model is that the conditional variances of $Xi \mid \Theta$ are similar. To address these problems, Bühlmann and Straub (1970) presented a generalized version of the Bühlmann model [Klugman et al., 1998].
Under the assumptions in the Bühlmann model, for $i = 1, 2, \ldots, n + 1$, the conditional mean and variance of Xi expressed in below

$$E[X_i| \Theta = \theta] = \mu\ (\theta) \tag{4.11}$$

$$Var\ [X_i| \Theta = \theta] = \vartheta\ (\theta)\ /\ m_i \tag{4.12}$$

m_i is the value of a risk exposure known to individuals during the i'th policy period. The same definitions can be used in the Bühlmann-Straub model as it is in the Bühlmann model. Here, μ (θ) is the hypothetical mean and ϑ (θ) is the process variance,

$$\mu = E[\mu\ (\theta)] \tag{4.13}$$

$$\vartheta = E[\ \vartheta\ (\theta)] \tag{4.14}$$

$$a = Var\ [\mu\ (\theta)] \tag{4.15}$$

is expressed in above.

μ: Expected value of hypothetical averages

ϑ: expected value of process variance

a: variance of hypothetical averages

the averages, variance and covariance of X_i can be obtained as follows.

$$E[X_i] = E[E(X_i| \Theta = \theta)] = E[\mu(\theta)] = \mu \qquad i = 1, 2, \ldots, n \tag{4.16}$$

$$\begin{aligned} Var[X_i] &= E[Var(X_i| \Theta = \theta)] + Var[E(X_i| \Theta = \theta)] \\ &= E[\vartheta(\theta)] / m_i] + Var[\mu(\theta)] \\ &= \vartheta/m_i + \alpha \end{aligned} \tag{4.18}$$

and

$$\begin{aligned} Kov(X_i, X_s) &= E(X_i, X_s) - E(X_i)E(X_s) \qquad s \neq i \quad s, i = 1, 2, 3, \ldots, n \\ &= E[E(X_i X_s| \Theta = \theta)] - \mu^2 \\ &= E[E(X_i| \Theta = \theta) E(X_s| \Theta = \theta)] - \{E[\mu(\theta)]\}^2 \\ &= \{E[\mu(\theta)]^2\} - \{E[\mu(\theta)]\}^2 \\ &= Var[\mu(\theta)] \\ &= \alpha \end{aligned} \tag{4.19}$$

The total number of units for the policy periods exposed to labor is $m = \sum_{i=1}^{n} mi$.

Using equation (4.16) for this model;

$$E[\mu_{n+1}(\theta)] = \tilde{a}_0 + \sum_{i=1}^{n} \tilde{a}_i E(X_i) \tag{4.20}$$

$$\mu = \tilde{a}_0 + \sum_{i=1}^{n} \tilde{a}_i \rightarrow \sum_{i=1}^{n} \tilde{a}_i = 1 - \tilde{a}_0/\mu \tag{4.21}$$

the above results are obtained.

For $s, i = 1, 2, 3, \ldots, n$;

Equality of $$Kov(X_s, X_{n+1}) = \sum_{i=1}^{n} \tilde{a}_i \, Kov(X_s, X_i) \tag{4.22}$$

Turns into;

$$\alpha = \sum_{i=1}^{n} \tilde{a}_i \alpha + \tilde{a}_s(\alpha + \vartheta/m_s) = \sum_{i=1}^{n} \tilde{a}_i \alpha + \vartheta \tilde{a}_s/m_s \quad (4.23)$$

When the obtained equality is solved according to $\tilde{a}_s$;; equity is as below;

$$\tilde{a}_s = \frac{\alpha}{v} m_s \left(1 - \sum_{i=1}^{n} \tilde{a}\,i\right) = \frac{\alpha}{v}\frac{\tilde{a}_0}{\mu} m_s \quad s,i = 1,2,\ldots,n \quad (4.24)$$

From 4.24 equation; 4.25 is obtained.

$$\sum_{i=1}^{n} \tilde{a}_i = 1 - \frac{\tilde{a}_0}{\mu} = \frac{\alpha}{v}\frac{\tilde{a}_0}{\mu} \sum_{i=1}^{n} m_s = \frac{\alpha}{v}\frac{\tilde{a}_0}{\mu} m \quad (4.25)$$

Here $\tilde{a}_0$,

$$\tilde{a}_0 = \frac{\mu}{1 + \frac{ma}{v}} = \frac{v/\alpha}{m_{+}v/\alpha}\,\mu \quad (4.26)$$

$$\tilde{a}_0 + \sum_{i=1}^{n} a_i X_i = \frac{v/\alpha}{m_{+}v/\alpha}\mu + \sum_{i=1}^{n} \frac{m_i}{m + v/\alpha} X_i \quad (4.27)$$

$$= (1-Z)\mu + Z\bar{X}$$

is obtained. Credibility factor and $\bar{X}$ are calculated as follows.

$$Z = \frac{m}{m+k} \text{ ve } \bar{X} = \sum_{i\equiv 1}^{n} \frac{m_i}{m} x_i \quad (4.28)$$

Credibility factor of Bühlmann-Straub Z, depends on m. Also $\bar{X}$, is the weighted average of X_i. These weights are proportional to m_i's. In the group formed by policyholders, X_i represents the average loss of the group member exposed by the insurance holders in the group formed by the policyholders. It can be interpreted that $m_i X_i$ expresses the total loss during the period of the group. $\overline{X}$, represents the overall average loss of each group member over a period of $\bar{n}$ period.

In conclusion, Bühlman-Straub credibility model; summarized in the form of

$$E(X_{n+1}) = (1-Z)\mu + Z\overline{X}, \quad Z = \frac{m}{m+k}, \; 0 \leq Z \leq 1 \qquad \text{(Ebegil 2007)}$$

In some cases, the value of $\tilde{a}$ gives negative results. As a result, when the premium value is calculated, the Z credibility factor is set to zero. This means that the credibility premium estimate given in above is based on the general average of all insured persons, with zero weighting on their experience. In other words, the insured's own experience is kept out of business while the premium estimate is being calculated. If the insured's damage experience is higher than that of other insureds, only the general average of the premium estimates will be in favor of the other insurers, while it will be in favor of the insured.

CHAPTER V
DATA ANALYSIS AND BUHLMANN-STRAUB MODELING

In this chapter; based on the credibility history and the Bühlmann-Straub credibility model as described in previous chapters; the premium data will be calculated using the actual data set for the traffic insurances and the damage modeling steps will be shown.

5.1 MODELING AND CALCULATION OF DAMAGE AMOUNT BY USING BUHLMANN – STRAUB MODEL

First, in 1930 being implemented in the UK and mainly traffic accidents involved a vehicle's damage to the opposite side of the defect rate of compensation that liability insurance is insurance group, Highways with the full name of Compulsory Liability Insurance is implemented since 1953 in Turkey. It is prohibited by law from the traffic of vehicles that have not been compulsory traffic insurance. It is aimed to prevent victims who will be suffered from traffic accidents by legal arrangements and measures made by lawmakers (Baykal et al. 2016)

Traffic insurance; is a legal type of insurance that covers the legal liability of the car owner in accordance with the Road Traffic Law up to compulsory insurance limits in the event of the death, injury or damage of any person during the use of the vehicle. (GençkayaSigorta site, Sigortam.net site)

According to the Road Traffic Law, every vehicle that comes to traffic has to have this insurance and renew it every year before it expires. Otherwise, the vehicle is prevented from moving traffic. Traffic Insurance, is valid within the borders of the Republic of Turkey. (GençkayaSigorta site, Sigortam.net site)

The limits on the compulsory traffic insurance vary depending on the type of damage caused by traffic accidents. In the case of any loss of life, the amount to be given to the relatives of the deceased is different from the amount to be paid for hospital expenses in case of injury. In the same way, the amount to be paid differs only in case of material damage. In such cases, the amount to be paid varies from year to year, and thus the premiums to be paid each year also change [Sigortam.net site].

Due to these changes, the premium calculation period for 2018 will be reviewed and the process of using the Bühlmann-Straub model for this review will be carried out.

We supplied the data which use for Bühlmann – Straub Credibility premium forecasting operations from TSB. 2014, 2015, 2016 and 2017 traffic insurance data will be used to estimate the amount of damage to the 2018 traffic insurance. [TSB]

Data received from the TSB was seen as described for 15 different vehicle types examined when the traffic insurance. These types of vehicles are as follows; and calculations will be made on the following topics in this study.

- ✓ Automobile
- ✓ Taxi
- ✓ Minibus (10-17 seats including driver)
- ✓ Bus (18-30 seats including driver)
- ✓ Bus (31 and over seat including driver)
- ✓ Van
- ✓ Truck
- ✓ Heavy construction equipment
- ✓ Tractor
- ✓ Trailer
- ✓ Motorcycle and Freight Motorcycle
- ✓ Tanker
- ✓ Towing vehicle
- ✓ Special Vehicle
- ✓ Agricultural Machine

The following is an edited version of the raw data received from Turkey Insurance Association. After the regulated data; The total values of 4 years were calculated and the average values were calculated by using these total values. In this case, the values shown by the charts are as follows:

- ✓ Chart 1; Shows the amount of total damage / compensation paid to vehicle type-based records in 2014, 2015, 2016, 2017.
- ✓ Chart 2; Table 2; In 2014, 2015, 2016, 2017 vehicle type of -based records shows total insured / policy the number.
- ✓ Chart 3; In 2014, 2015, 2016, 2017 the amount of premiums on vehicle type-based records.
- ✓ Chart 4; In 2014, 2015, 2016, 2017, vehicle type-based records will be used; dividing the total damages by the total number of vehicles.
- ✓ Table 5; It shows the number of files / damage opened as a result of the damage to the vehicle type based records in 2014, 2015, 2016, 2017.

Chart 5.1: Annual Compensation / Damage Amount According to Vehicle Type

	COMPENSATION (AMOUNT OF DAMAGE)					
Vehicle Type	2014	2015	2016	2017	TOTAL	AVERAGE
1-Automobil	1.712.471.080	2.145.380.978	2.796.801.067	3.210.947.941	9.865.601.066	2.466.400.266,51
2-Taxi	52.798.476	65.144.919	79.905.568	105.340.504	303.189.466	75.797.366,42
3-Minibus (10-17 seats including driver)	140.819.912	167.067.788	223.724.037	249.830.365	781.442.101	195.360.525,32
4-Bus (18-30 seats including driver)	94.222.826	103.322.520	128.396.643	140.943.730	466.885.719	116.721.429,71
5-Bus (31 and over seat including driver)	51.440.350	59.062.536	62.934.802	74.845.049	248.282.737	62.070.684,27
6-Van	752.276.587	890.274.186	1.101.741.695	1.233.943.022	3.978.235.491	994.558.872,68
7-Truck	276.553.496	309.432.491	365.984.457	365.219.846	1.317.190.290	329.297.572,46
8-Heavy construction equipment	7.534.527	9.064.755	11.095.494	13.484.805	41.179.580	10.294.895,04
9-Tractor	28.460.846	41.569.853	57.600.949	59.114.162	186.745.810	46.686.452,53
10-Trailer	285.802	151.220	156.956	256.055	850.033	212.508,16
11-Motorcycle and Freight Motorcycle	71.167.249	98.831.439	131.248.773	138.332.783	439.580.244	109.895.060,97
12-Tanker	8.464.812	10.593.094	12.682.492	12.168.831	43.909.229	10.977.307,23
13-Towing vehicle	151.605.753	188.911.421	240.564.396	285.306.688	866.388.259	216.597.064,70
14-Special Vehicle	17.417.372	22.233.469	27.274.374	31.892.901	98.818.115	24.704.528,76
15-Agricultural Machine	155.458	68.254	179.819	222.185	625.716	156.428,96
TOTAL	3.365.674.544	4.111.108.922	5.240.291.521	5.921.848.867	18.638.923.855	

Türkiye Sigorta Birliği (https://www.tsb.org.tr/resmi-istatistikler.aspx?pageID=909, 2018)

Chart 5.2: Annual Number of Insured / Policy Accoding to Vehicle Type

	NUMBER OF POLICIES (TOTAL NUMBER OF INSURED PERSONS)					
Vehicle Type	2014	2015	2016	2017	TOTAL	AVERAGE
1-Automobil	9.122.624	9.415.487	10.261.689	11.006.061	39.805.860	9.951.465,07
2-Taxi	74.820	75.127	82.242	73.008	305.197	76.299,28
3-Minibus (10-17 seats including driver)	348.359	356.917	369.527	376.658	1.451.460	362.865,12
4-Bus (18-30 seats including driver)	139.187	135.983	130.307	128.303	533.780	133.444,92
5-Bus (31 and over seat including driver)	36.460	33.465	28.723	27.915	126.564	31.641,09
6-Van	2.730.079	2.782.531	2.964.434	3.142.225	11.619.269	2.904.817,18
7-Truck	388.451	394.103	393.402	379.418	1.555.374	388.843,46
8-Heavy construction equipment	51.600	55.901	61.778	56.593	225.872	56.468,10
9-Tractor	785.257	860.742	941.433	988.529	3.575.961	893.990,30
10-Trailer	3.763	989	809	1.164	6.726	1.681,44
11-Motorcycle and Freight Motorcycle	1.126.335	1.141.744	996.881	1.064.059	4.329.019	1.082.254,76
12-Tanker	16.125	19.026	19.150	19.628	73.928	18.482,04
13-Towing vehicle	211.831	217.677	213.695	212.179	855.382	213.845,54
14-Special Vehicle	26.642	30.330	35.065	44.328	136.364	34.091,09
15-Agricultural Machine	1.403	2.411	3.277	4.046	11.137	2.784,19
TOTAL	15.062.936	15.522.432	16.502.412	17.524.114	64.611.894	

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Chart 5.3: Annual Premiums Calculated by Vehicle Weight

	PREMIUM					
Vehicle Type	2014	2015	2016	2017	TOTAL	AVERAGE
1-Automobil	2.180.054.027	3.233.391.839	6.439.814.858	5.926.271.978	17.779.532.703	4.444.883.176
2-Taxi	85.459.092	98.386.891	183.906.139	138.127.562	505.879.684	126.469.921
3-Minibus (10-17 seats including driver)	206.096.931	293.772.361	512.089.458	428.732.214	1.440.690.964	360.172.741
4-Bus (18-30 seats including driver)	137.972.347	180.616.568	288.996.140	224.444.405	832.029.460	208.007.365
5-Bus (31 and over seat including driver)	108.377.840	119.376.192	157.370.260	117.472.548	502.596.840	125.649.210
6-Van	973.693.329	1.339.306.768	2.508.902.330	2.289.176.749	7.111.079.176	1.777.769.794
7-Truck	453.601.760	550.380.364	801.687.209	673.603.974	2.479.273.307	619.818.327
8-Heavy construction equipment	24.169.784	28.032.277	36.897.158	31.367.712	120.466.931	30.116.733
9-Tractor	68.151.254	95.251.799	178.691.426	133.773.083	475.867.562	118.966.891
10-Trailer	890.284	315.010	425.334	1.033.271	2.663.899	665.975
11-Motorcycle and Freight Motorcycle	115.439.411	156.317.997	326.021.197	266.285.303	864.063.908	216.015.977
12-Tanker	16.586.578	22.427.145	32.554.201	22.179.818	93.747.742	23.436.935
13-Towing vehicle	399.457.531	497.506.420	720.858.766	595.156.129	2.212.978.847	553.244.712
14-Special Vehicle	32.171.828	48.346.451	77.169.196	72.247.195	229.934.670	57.483.668
15-Agricultural Machine	215.835	444.015	847.906	720.868	2.228.624	557.156
TOTAL	4.802.337.832	6.663.872.096	12.266.231.579	10.920.592.810	34.653.034.317	

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Chart 5.4: Annual Average Amount of Damage Per Insured

Vehicle Type	AVERAGE DAMAGE AMOUNT FOR EACH INSURED					
	2014	2015	2016	2017	TOTAL	AVERAGE
1-Automobil	187,72	227,86	272,55	291,74	979,87	244,97
2-Taxi	705,67	867,13	971,59	1.442,87	3987,26	996,82
3-Minibus (10-17 seats including driver)	404,24	468,09	605,43	663,28	2141,04	535,26
4-Bus (18-30 seats including driver)	676,95	759,82	985,34	1.098,52	3520,63	880,16
5-Bus (31 and over seat including driver)	1.410,85	1.764,89	2.191,06	2.681,17	8047,97	2.011,99
6-Van	275,55	319,95	371,65	392,70	1359,85	339,96
7-Truck	711,94	785,16	930,31	962,58	3389,98	847,50
8-Heavy construction equipment	146,02	162,16	179,60	238,28	726,05	181,51
9-Tractor	36,24	48,30	61,18	59,80	205,52	51,38
10-Trailer	75,95	152,89	193,90	219,91	642,66	160,66
11-Motorcycle and Freight Motorcycle	63,18	86,56	131,66	130,00	411,41	102,85
12-Tanker	524,95	556,77	662,29	619,99	2363,99	591,00
13-Towing vehicle	715,69	867,85	1.125,74	1.344,65	4053,93	1.013,48
14-Special Vehicle	653,76	733,06	777,82	719,48	2884,12	721,03
15-Agricultural Machine	110,80	28,31	54,87	54,92	248,90	62,23

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Chart 5.5: Annual Number of Damage /Damage File by Vehicle Type

Vehicle Type	NUMBER OF DAMAGED FILES (TOTAL DAMAGED NUMBER)					
	2014	2015	2016	2017	TOTAL	AVERAGE
1-Automobil	616.496	647.194	669.722	726.441	2.659.853	664.963,25
2-Taxi	20.938	22.574	22.294	23.037	88.843	22.210,75
3-Minibus (10-17 seats including driver)	41.673	45.134	45.961	53.178	185.946	46.486,50
4-Bus (18-30 seats including driver)	32.662	30.324	27.603	29.185	119.774	29.943,50
5-Bus (31 and over seat including driver)	11.319	11.086	10.312	9.859	42.576	10.644,00
6-Van	283.719	288.638	276.009	300.365	1.148.731	287.182,75
7-Truck	74.492	78.232	73.895	77.856	304.475	76.118,75
8-Heavy construction equipment	2.511	2.714	2.945	4.310	12.480	3.120,00
9-Tractor	5.078	5.737	5.869	6.344	23.028	5.757,00
10-Trailer	74	71	43	42	230	57,50
11-Motorcycle and Freight Motorcycle	21.030	22.135	21.177	21.930	86.272	21.568,00
12-Tanker	1.744	2.068	2.111	2.170	8.093	2.023,25
13-Towing vehicle	41.482	45.587	46.118	48.845	182.032	45.508,00
14-Special Vehicle	4.616	4.780	4.648	5.157	19.201	4.800,25
15-Agricultural Machine	21	20	24	58	123	30,75
TOTAL	1.157.855	1.206.294	1.208.731	1.308.777	4.881.657	

Türkiye Sigorta Birliği (https://www.tsb.org.tr/resmi-istatistikler.aspx?pageID=909, 2018)

In this study we used the data of the last 4 years to model the 2018 damage amount or the premium payment vehicle type that will be requested from the insured. For modeling; The Bühlmann-Straub credibility model will lead us. Our aim is to model the amount of damage for 15 vehicle types; in this model we will calculate by using the average damage amount data (Chart 5.4).

In this context, the calculated parameter values for the Bühlmann-Straub model will be estimated as follows.

$$\hat{\mu} = \bar{X} = \frac{1}{m}\sum_{i=1}^{15}\sum_{j=1}^{4} m_{ij}X_{ij} = 288{,}47$$

$\bar{X} = 288{,}47$

$$\bar{v} = \frac{\sum_{i=1}^{15}\sum_{j=1}^{4} m_{ij}\left(x_{ij} - \bar{x}_i\right)^2}{\sum_{i=1}^{15} t_i - 1}$$

$\bar{v}$ =306.509.853.774.467.000.000.000,00

$$\hat{\alpha} = \frac{\sum_{i=1}^{15} m_i(\bar{x_i} - \bar{x})^2 - \hat{v}(n-1)}{m - \frac{1}{m}\sum_{i=1}^{15} m_i^2}$$

$\hat{\alpha} = 6.671.953.336.900.620.000{,}00$

After these calculations, the value of $\widehat{\boldsymbol{k}}$ is calculated.

$$\hat{k} = \frac{\bar{v}}{\bar{\alpha}}$$

$\hat{k}$= 45.940,05

Credibility factor is;

$$\hat{Z}_i = \frac{m_i}{m_i + \hat{k}}$$

The estimated of credibility factor ($\hat{Z}_i$) and average damage amount ($\bar{X}_i$) for 15 vehicles *(i = 1, 2, ..., 15)* are shown in the table as follows;

Figure 5.1: The Premium Value Table Calculated by The Average Value

$\hat{z}_i$		$\bar{x}_i$	
$\hat{z}_1 =$	0,99884723	$\bar{x}_1 =$	244,97
$\hat{z}_2 =$	0,86916780	$\bar{x}_2 =$	996,82
$\hat{z}_3 =$	0,96932013	$\bar{x}_3 =$	532,26
$\hat{z}_4 =$	0,92075473	$\bar{x}_4 =$	880,16
$\hat{z}_5 =$	0,73368768	$\bar{x}_5 =$	2.011,99
$\hat{z}_6 =$	0,99606179	$\bar{x}_6 =$	339,96
$\hat{z}_7 =$	0,97131103	$\bar{x}_7 =$	847,50
$\hat{z}_8 =$	0,83098623	$\bar{x}_8 =$	181,51
$\hat{z}_9 =$	0,98731604	$\bar{x}_9 =$	51,38
$\hat{z}_{10} =$	0,12770670	$\bar{x}_{10} =$	160,66
$\hat{z}_{11} =$	0,98949932	$\bar{x}_{11} =$	102,85
$\hat{z}_{12} =$	0,61674541	$\bar{x}_{12} =$	591,00
$\hat{z}_{13} =$	0,94903038	$\bar{x}_{13} =$	1.013,48
$\hat{z}_{14} =$	0,74800364	$\bar{x}_{14} =$	721,03
$\hat{z}_{15} =$	0,19511874	$\bar{x}_{15} =$	62,23

The credibility factor and the average of the damage were calculated. Finally, the premium for each vehicle type will be calculated with the Bühlmann-Straub credibility formula. This formula is as follows. Chart 6 shows the calculated premium values for each vehicle type.

$$C_i = \hat{Z}_i \bar{X}_i + (1 - \hat{Z}_i)\hat{\mu}$$

Figure 5.2: Damage Amount for 2018 by Vehicle Type

Vehicle Type	Premium of 2018 c_i	
Automobil	$c_1 =$	245,02
Taxi	$c_2 =$	904,14
Minibus (10-17 seats including driver)	$c_3 =$	527,69
Bus (18-30 seats including driver)	$c_4 =$	833,27
Bus (31 and over seat including driver)	$c_5 =$	1.553,00
Van	$c_6 =$	339,76
Truck	$c_7 =$	831,46
Heavy construction equipment	$c_8 =$	199,59
Tractor	$c_9 =$	54,39
Trailer	$c_{10} =$	272,15
Motorcycle and Freight Motorcycle	$c_{11} =$	104,80
Tanker	$c_{12} =$	475,05
Towing vehicle	$c_{13} =$	976,53
Special Vehicle	$c_{14} =$	612,03
Agricultural Machine	$c_{15} =$	244,33

When the calculated damage amounts are examined, the biggest damage amounts are determined for Bus (31 and more seats including Driver), Towing Vehicle and Taxi respectively.

In the credibility method; As can be seen from the Bühlmann-Straub formula of credibility; the insurer's own experience value and the experience value of all insurers affect the calculated premium value. At this point, it is important to determine the Z factor when making a premium account. Z factor close to 0 or 0 will determine the weight of the insured experience as 0; the premium value of the insured shall be equal to the general average $\mu = 288.47$. The calculated value will be equal to the overall experience of the insured. Since the probability of each insured being damaged is different; the insured who is required to pay a premium according to the general average value will pay the premium more even though it makes little damage and this situation will be against the insured. Otherwise; will result in a low premium payment for someone who is excessively damaged.

Given the calculated values, the premium value for the van was calculated as 339,76. If the premium is given according to the average value, the insured will pay 288.47. In this case the insured will pay a deficit of TL 51.29. In the other case, the premium calculated for the automobile is 245.02. If the premium is paid according to the average value, the insured will pay TL 288.47 and pay TL 43,45 more than the required amount to pay.

On the other hand, the credibility factor Z, is taken as 1; is not included in the premium account of the insured's general experience.

In both cases, determining the Z credibility factor is important for the premium account. The establishment of a Z factor which consists of the insurer's own experience and the combination of experiences of all insurers; allowing for the removal of favorable or unfavorable circumstances of the insured.

Average values of the past 4 years for the above calculations is used instead of $\bar{X}_i$. The following new independent averages are recalculated by including in the calculations for each vehicle type.

Figure 5.3: Damage Amount for 2018 by Vehicle Type With Random Datas

$\hat{z}_i$		$\bar{x}_i$	
$\hat{z}_1 =$	**0,99884723**	$\bar{x}_1 =$	**295,76**
$\hat{z}_2 =$	**0,86916780**	$\bar{x}_2 =$	**1.665,75**
$\hat{z}_3 =$	**0,96932013**	$\bar{x}_3 =$	**712,30**
$\hat{z}_4 =$	**0,92075473**	$\bar{x}_4 =$	**5.252,88**
$\hat{z}_5 =$	**0,73368768**	$\bar{x}_5 =$	**3.360,28**
$\hat{z}_6 =$	**0,99606179**	$\bar{x}_6 =$	**413,08**
$\hat{z}_7 =$	**0,97131103**	$\bar{x}_7 =$	**985,56**
$\hat{z}_8 =$	**0,83098623**	$\bar{x}_8 =$	**249,66**
$\hat{z}_9 =$	**0,98731604**	$\bar{x}_9 =$	**60,13**
$\hat{z}_{10} =$	**0,12770670**	$\bar{x}_{10} =$	**241,77**
$\hat{z}_{11} =$	**0,98949932**	$\bar{x}_{11} =$	**125,12**
$\hat{z}_{12} =$	**0,61674541**	$\bar{x}_{12} =$	**641,45**
$\hat{z}_{13} =$	**0,94903038**	$\bar{x}_{13} =$	**1.337,78**
$\hat{z}_{14} =$	**0,74800364**	$\bar{x}_{14} =$	**745,55**
$\hat{z}_{15} =$	**0,19511874**	$\bar{x}_{15} =$	**57,35**

The new values are calculated in the Bühlmann-Straub credibility formula. Chart 7 shows the calculated premium values for each vehicle type.

Figure 5.4: New Premium Amount for 2018 by Vehicle Type

Vehicle Type	Premium of 2018 c_i	
Automobil	$c_1 =$	295,75
Taxi	$c_2 =$	1.485,56
Minibus (10-17 seats including driver)	$c_3 =$	699,29
Bus (18-30 seats including driver)	$c_4 =$	4.859,47
Bus (31 and over seat including driver)	$c_5 =$	2.542,21
Van	$c_6 =$	412,59
Truck	$c_7 =$	965,55
Heavy construction equipment	$c_8 =$	256,22
Tractor	$c_9 =$	63,02
Trailer	$c_{10} =$	282,51
Motorcycle and Freight Motorcycle	$c_{11} =$	126,83
Tanker	$c_{12} =$	506,17
Towing vehicle	$c_{13} =$	1.284,29
Special Vehicle	$c_{14} =$	630,36
Agricultural Machine	$c_{15} =$	243,37

When the damage amounts calculated with the new data are examined, the biggest damage amounts are determined for Bus (18 - 30 seats including driver), Bus (31 and over seat including driver) and taxi respectively.

Figure 5.5: Comparison Datas

Average Values			Random Values		
Vehicle Type	Premium of 2018 c_i		Vehicle Type	Premium of 2018 c_i	
Automobil	$c_1 =$	245,02	Automobil	$c_1 =$	295,75
Taxi	$c_2 =$	904,14	Taxi	$c_2 =$	1.485,56
Minibus (10-17 seats including driver)	$c_3 =$	527,69	Minibus (10-17 seats including driver)	$c_3 =$	699,29
Bus (18-30 seats including driver)	$c_4 =$	833,27	Bus (18-30 seats including driver)	$c_4 =$	4.859,47
Bus (31 and over seat including driver)	$c_5 =$	1.553,00	Bus (31 and over seat including driver)	$c_5 =$	2.542,21
Van	$c_6 =$	339,76	Van	$c_6 =$	412,59
Truck	$c_7 =$	831,46	Truck	$c_7 =$	965,55
Heavy construction equipment	$c_8 =$	199,59	Heavy construction equipment	$c_8 =$	256,22
Tractor	$c_9 =$	54,39	Tractor	$c_9 =$	63,02
Trailer	$c_{10} =$	272,15	Trailer	$c_{10} =$	282,51
Motorcycle and Freight Motorcycle	$c_{11} =$	104,80	Motorcycle and Freight Motorcycle	$c_{11} =$	126,83
Tanker	$c_{12} =$	475,05	Tanker	$c_{12} =$	506,17
Towing vehicle	$c_{13} =$	976,53	Towing vehicle	$c_{13} =$	1.284,29
Special Vehicle	$c_{14} =$	612,03	Special Vehicle	$c_{14} =$	630,36
Agricultural Machine	$c_{15} =$	244,33	Agricultural Machine	$c_{15} =$	243,37

As can be understood from these calculations; the values taken as the average damage data per insured have a considerable effect on the premium value calculation. Also, as mentioned above, the calculation of the Z credibility factor and $\bar{X}_i$ are important for the amount of premium payout.

Figure 5.6: Comparison Datas with 2017 Premium

Comparison of Premium				
Vehicle Type	2017	4 Years Average (2014-2017)	Calculated Premium of 2018 with Average Values	Calculated Premium of 2018 with Random Values
Automobil	291,74	244,97	245,02	295,75
Taxi	1.442,87	996,82	904,14	1.485,56
Minibus (10-17 seats including driver)	663,28	535,26	527,69	699,29
Bus (18-30 seats including driver)	1.098,52	880,16	833,27	4.859,47
Bus (31 and over seat including driver)	2.681,17	2.011,99	1.553,00	2.542,21
Van	392,70	339,96	339,76	412,59
Truck	962,58	847,50	831,46	965,55
Heavy construction equipment	238,28	181,51	199,59	256,22
Tractor	59,80	51,38	54,39	63,02
Trailer	219,91	160,66	272,15	282,51
Motorcycle and Freight Motorcycle	130,00	102,85	104,80	126,83
Tanker	619,99	591,00	475,05	506,17
Towing vehicle	1.344,65	1.013,48	976,53	1.284,29
Special Vehicle	719,48	721,03	612,03	630,36
Agricultural Machine	54,92	62,23	244,33	243,37
			μ = 288.47	μ =339,76

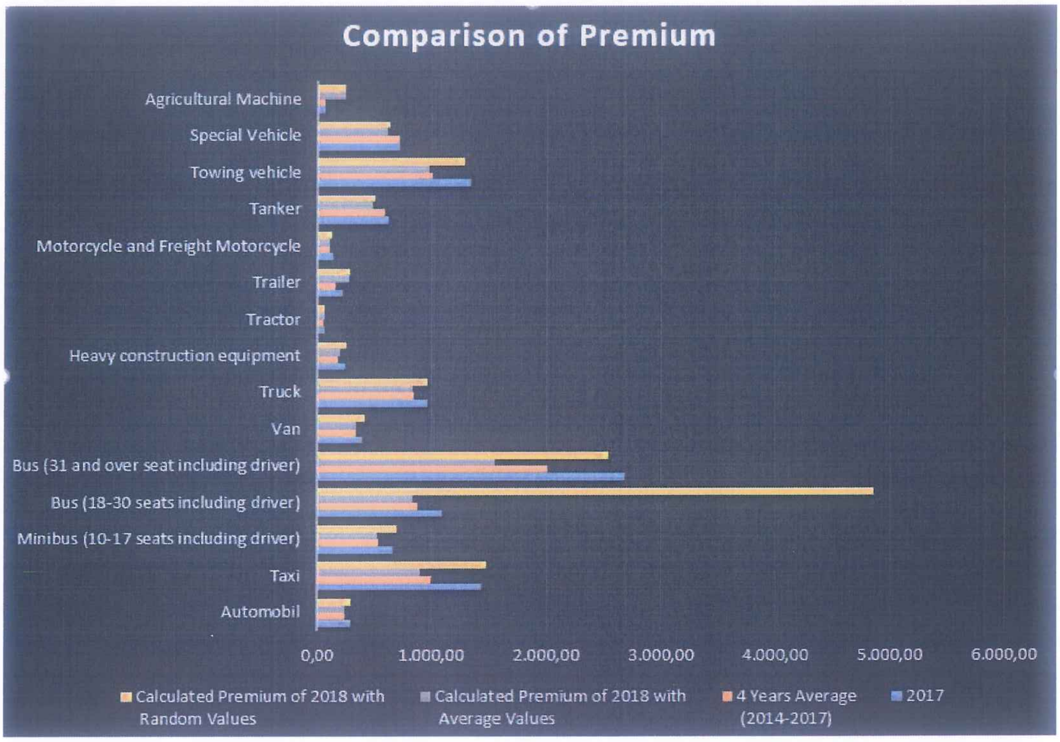
When looking at the table above, premiums for automobile are 291,74 in 2017, while the average of 2014-2017 is 244,97. The credibility premium calculated using the averages was found to be 245,02, while the μ = 288,47 versus the 2018 premium. On the other hand; 2018 premium value for automobile using random data; was calculated as 295,75 when μ = 339,76. For both cases the automobile premium is calculated less than μ value.

The premium for taxi in 2017 is 1.442,87, while the average of 2014-2017 is 996,82. The credibility calculated using the averages was found to be 904,14, while μ = 288,47 versus the 2018 premium value. On the other hand; premium value for taxi using random data; was calculated as 1.485,56 when μ = 339,76. For both cases, the taxi premium is calculated more than the μ value.

The premium for the minibus is 663,28 in 2017, while the average for 2014-2017 is 535,26. According to the credibility premium calculated using the averages; μ = 288.47 while the 2018 premium value for the minibus was found to be 527,69. On the other hand; premium value for minibus using random data; was calculated as

699,29 when μ = 339,76. For both cases, the minibüs premium is calculated more than the μ value.

Figure 5.7: Comparison Chart of Premium



When looking at the other values in the table; the premium values calculated by taking the average of 4 years are smaller than the premium values calculated by the randomly received data. For example, while the values calculated for motorcycles and agricultural machines are below μ values, premium values calculated for trucks and tankers are above μ value. In credibility premium account; the significance of the value as calculating Z credibility factor and the size of the sample set, $\bar{X}_i$, is important.

The selection of the credibility premium will benefit the insured for some types of vehicles; for some types of vehicles it will be at the expense of the insured.

CONCLUSION

The credibility is used to calculate weighted forecast values and the weighting is done with the Z credibility factor. The Z value is calculated by various credibility models; it is important to correctly determine the Z credibility factor with these models.

The estimated value of the hypothetical mean variance can be calculated negatively depending on the situations during the calculations. In this case it is correct to take $Z = 0$ if this value is negative. When $Z = 0$ is taken in the credibility formula, the premium estimate is determined only according to the general average of the insured. In other words, the insured's own experience premium is kept out of the forecast. At this point, if the insured's damage experience is higher than other insureds, getting the average will be against the opposite case in favor of the insured. This causes injustice among the insured. At this point, credibility calculation processes become important for ensuring your name.

To achieve fair results; the variance of the risk parameter is large, and the process variance value is small. In this case, if $Z \neq 0$, the premium calculation results will be fair for the insured.

Using the Bühlmann-Starub credibility formula, the damage amount and premium modeling procedures are examined and Z values are calculated to obtain fair results for the insured. Different premium values are obtained by taking the mean values and the random values $\bar{X}_i$ by the calculated Z values and using the average damage experience data according to the obtained data adds a positive value to the calculations for some cases while the premium value is calculated high in some cases as expected.

REFERENCES

Atanasiu, V., 2009, *Some extensions of the Bühlmann-Straub credibility formulae*, Blutenul Academiei De Stiinte A Republicii Moldova, 3(61):3-12.

Bailey, A., L., 1945, *Generalized Theory of Credibility*, Proc. Casualty Actuarial Soc. 32, 13-20.

Bailey, A. L., 1950, *Credibility Procedures, Laplace' s Generalization of Bayes Rule and the Combination of Collateral Knowledge with Observed Data.*, Proceedings of the Casualty Actuarial Society, 37: 7–23.

Baykal, K., & Bülbül, S. , 2016., *Türkiye'de Trafil Sigortalarında Optimal Ödül – Ceza Sistemi Tasarımı: İyi / Kötü Risk Modeli*, Finansal Araştırmalar ve Çalışmalar Dergisi, 8(14), 17-31.

Bühmann, H., 1967, Experience Rating and Credibility, *ASTIN Bulletin*, 4: 199–207 (1967).

Bühlmann, H. and Straub, E., 1970. Glaubwürdigkeit Für Schadensätze, *Mitt. Ver. Schweiz. Ver.*, 70(1): 111-133.

Campbell, M., 1986, An Integrated System For Estimating The Risk Premium Of Individual Car Models in Motor Insurance, *ASTIN Bulletin*,16(2), 165-183.

Dannenburg, D., 1995, Crossed Classification Credibility Models, *In. Trans. 25th Int. Congress of Actuaries*, 4: 1–36.

Dannenburg, D. R., Kaas R., Goovaerts M. J., 1996, Practical Actuarial Credibility Models, *University of Amsterdam Institute of Actuarial Science and Econometrics*, The Netherlands, 3-40.

De Vylder, F.,(1981), Practical Credibility Theory with Emphasis on Optimal Parameter Estimation. *ASTIN Bulletin*, 12: 115-132.

Ebegil, M., 2007, *Kredibilite Teorisinde Parametre Tahmini ve İstatistiksel Yaklaşım*, Doktora Tezi, Gazi Üniversitesi Fen Bilimleri Enstitüsü, Ankara, 48-78.

Ebegil M., 2006, *A Study to Examine Bülhmann-Straub Credibility Model in Generalized Linear Models*, A.U. Communications Series A1: Mathematics and Statistics, 55(2):9-16.

Erdal, A., 2014, *Klasik Kredibilite Modellerinde Kredibilite Faktörün İncelenmesi*, Yüksek Lisans Tezi, Gazi Üniversitesi Fen Bilimleri Enstitüsü, Ankara, 25-75.

Erdemir, Ö. K., & Sucu M., 2016, *Genelleştirilmiş Doğrusal Model Bileşenlerinin Kredibilite Üzerindeki Etkilerinin Araştırılması*, İstatistikçiler Dergisi: İstatistik ve Aktüerya, 9(1), 37-45.

Frees E. W., P. Wang, (2005), Credibility Using Copulas, *North American Actuarial Journal*, 9 (2), pp.31–48.].

Gau, W., Gangopadhyay, A., Han, Z., 2008, Interval Estimation of the Credibility Factor *Proceedings of the Casualty Actuarial Society,* 2(1): 71-84.

Goulet, V., 1998, Principles and Applications of Credibility Theory, *Journal of Actuarial Practice*, 6, 5-62.

Herzog, T.N. Introduction to Credibility Theory (3rd ed.), (1999), ACTEX Publications, Winsted.

Herzog, T., N., 1999, *Credibility Theory, ACTEX,* Third Edition, 273.

Kaas, R., Goovaerts, M., Dhaene, J., Denuit, M., 2008, *Modern Actuarial Risk Theory: Using R*, Springer Verlang Inc., Berlin, 381.

Karadağ, Ö. G. (2014), *Genelleştirilmiş Doğrusal Modeller Için Sınırlı Dalgalanmalı Kredibilite Yaklaşımı,* Yüksek Lisans Tezi, Hacettepe Üniversitesi Aktüerya Bilimleri Anabilim Dalı, Ankara, 113.

Klugman, S.A., 1992, Bayesian Statistics in Actuarial Science with Emphasis on Credibility, Kluwer, Boston.

Klugman, S.A., Panjer, H. H. and Willmot, G. E., 1998, "Loss Models From Data to Decisions 2nd ed.", *John Wiley & Sons*, New York, 515–600.

Klugman, S., Panjer, H., Wilmot, G., 2004, *Loss Models from Data to Decisions*, Willey, New York.

Kremer, E. (1994). Robust Credibility Via Robust Kalman Filtering. *ASTIN Bulletin,* 24(2): 221–233.

Longley-Cook, L.H. (1962), An Introduction to Credibility Theory, Proceeding of the Casualty Actuarial Society 49, 194-221.

Mahler, H.,C., Dean, C.G., 2005, *"Chapter 8: Credibility",* In Foundations of Casualty Actuarial Science, 4th Edition, Arlington, VA: Casualty Actuarial Society, 485-659.

Mert, M., 2006, *Kredibilite Prim Tahminlerinde Parametrik Olmayan Yaklaşımlar*, Doktora Tezi, Hacettepe Üniversitesi, İstatistik Ana Bilim Dalı, 83.

Mowbray, A. H. (1914). How extensive a payroll exposure is necessary to give a dependable pure premium. In Proceedings of the Casualty Actuarial society (Vol. 1, No. 1, pp. 24-30).

Norberg, R. (1986), Hierarchical Credibility: Analysis of a Random Effect Linear Model with Nested Classification. *Scandinavian Actuarial Journal,* 204-222.

Norberg, R., 2004 *Credibility Theory*, Encyclopedia of Actuarial Science,.

Schnieper, R. (1995). On the Estimation of the Credibility Factor: A Bayesian Approach. *ASTIN Bulletin,* 25(2): 137–151.

Sundt, B., Two Credibility Regression Approaches for the Classification of Passenger Cars in a Multiplicative Tariff, *ASTIN Bulletin*, 17(1), 42-70, 1987.

Şentürk, A., 2010, *Kredibilite Teorisinde Panel Veri Modelleri*, Yüksek Lisans Tezi, Hacettepe Üniversitesi Fen Bilimleri Enstitüsü, Ankara, 97,.

Wen, L., Deng, W., 2011 "The Credibiliyt Models With Equal Correlation Risks", *J Syst Sci Complex*, 24:532-539.

Whitney, A.,W., (1918), *The Theory Of Experience Rating*, Proc. of the Casualty Actuarial Society, 4, 274-292, 1918.

Young, V. R., (1997), *Credibility Using Semiparametric Models*, *ASTIN Bulletin*, 27(2): 273–285.

İnternet: Sigortam "Zorunlu Trafik Sigortası Nedir?"

https://www.sigortam.net/trafik-sigortasi/zorunlu-trafik-sigortasi-nedir (2018)

İnternet: GençKayaSigorta "Zorunlu Trafik Sigortası Nedir?"

http://genckayasigorta.com.tr/s-zorunlu-trafik-sigortasi-policesi-hesaplama.html (2018)

İnternet: ZORUNLU MALİ SORUMLULUK SİGORTASI (TRAFİK SİGORTASI) http://www.tazminathukuku.com/arastirma-yazilari/zorunlu-mali-sorumluluk-sigortasi-trafik-sigortasi.htm (2018)

İnternet: Türkiye Sigorta Birliği "Motorlu Taşıtlar İstatistikleri - Trafik"

https://www.tsb.org.tr/resmi-istatistikler.aspx?pageID=909 (2018)